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A SHORT HIGH POWER TELESCOPE

BY LT.-COL. J. WILLIAM GIFFORD, F.R.A.S.

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IN September 1913, during a conversation with my friend Mr E. M. Nelson, he remarked that one of the Schott boro-silicate crowns, type O. 144, used in conjunction with a suitable flint glass would develop approximately equal radii on both surfaces, and on running through the figures a few weeks later this proved to be correct.

In 1915 I was engaged in some optical work for the War Department and received an enquiry for a short telescope having an equivalent focus of about $8\frac{1}{2}$ inches. The little $f/7$ telescope now to be described is the result of this and of a visit to the front early in 1916. It was intended for the pocket of the "British Warm" and for use as a trench telescope on an ordinary sandbag, than which there is no better support.

The O.G. of this $f/7$ telescope is a typical one-radius doublet whose posterior surface is plane. The optical glasses employed were:

Schott's S. 614 (type O. 144) boro-silicate crown.
Chance's C. 629 (type 384) heavy flint.

Of these the corresponding refractive indices for line A (the maximum visual point) and the dispersions were respectively:

$$\begin{matrix} A \\ 1{\cdot}51132 \\ 1{\cdot}61472 \end{matrix} \qquad \left.\begin{matrix} \delta\mu = 0{\cdot}00807 \\ \delta\mu' = 0{\cdot}01656 \end{matrix}\right\} \frac{\delta\mu}{\delta\mu'} = 0{\cdot}4873.$$

This therefore is an overcorrected combination.

The first step taken was to reverse the order, putting the flint first, and in this way to calculate an achromatic plano-convex doublet of 8″·50 equivalent focus,

noting how nearly equal to one another the radii of curvature work out. Of course, provided that we can find a flint and a crown such that the dispersive power of the flint is exactly twice that of the crown, we shall find our radii automatically equal, and all that remains to be done for our one-radius doublet is to turn it round so that the crown component is leading, and adjust the thicknesses until all the spherical aberration has vanished. Coma then is moderate. But such a balance would exact both erector and eyepiece being achromatic, with the attendant heavy cost. Now let us see what we can do with the two meltings of boro-silicate crown and heavy flint already referred to.

Proceeding in the ordinary way we now construct an achromatic cemented doublet with a plane flint surface leading, for a focus of unity but without thickness, and find the radii of curvature work out as follows:

$$R_0 = \infty, \qquad R_2 = R_4 = 0{\cdot}4345315, \qquad R_6 = -0{\cdot}4130353.$$

These radii were then multiplied by the focal length required (8″·50) and the thicknesses put in. These were respectively:

$$t_1 = 0{\cdot}075, \qquad t_3\,(\text{cement}) = 0{\cdot}000, \qquad t_5 = 0{\cdot}250.$$

A trigonometrical trace for paraxial rays was then carried out in order to ascertain the variation of R_6 due to thickness. The resulting radii were:

$$R_0 = \infty, \qquad R_2 = R_4 = 3{\cdot}693518, \qquad R_6 = -3{\cdot}527058.$$

Turning the doublet round and making traces for lines *C* and *F* we find the foci differ by 0·00217 undercorrected (I omit the tedious figures). Now if we can find a pair of tools of approximately 3″·53 radius, it will be evident that again adjusting thickness for spherical aberration, we can quite nearly satisfy our requirements by adopting this for both radii, but our doublet will then be perceptibly overcorrected for colour*. Indeed by further traces the outstanding error was ascertained to amount to − 0·005178, overcorrected, and thus the former undercorrection of 0·002176 has been wiped out.

And in this case practical experiment proves that proportions have been taken which are just compensated by an ordinary non-achromatic Dollond erecting eyepiece†, and thus a very perfect achromatism can be achieved for this telescope.

Personally I prefer a partially achromatic erector used in combination with an ordinary Huygenian eyepiece, since a more extended field of view is available. It was in this form that the *f*/7 telescope was originally made, and in this connection it may be well to describe exactly the form of eyepiece then adopted.

In continuation of the idea of simplest construction it was decided to make the achromatic doublets of which the erector was composed, one-radius doublets, using the same glass as had already been made use of for the O.G. It was first required to find a pair of tools of such radius of curvature that when two of the doublets were placed with their crown components facing one another at such a separation that the principal planes of the combination were in coincidence (Fig. 1), the combined focus should be equal approximately to 0″·753. Messrs Hilger, Ltd.,

* Such a tool was found for me by Messrs A. Hilger, Ltd.

† See Heath's *Optics*, 2nd edition (1895), par. 262.

again found me a tool which sufficiently fell in with these requirements. This tool had a radius of o″·527. Putting it into use we get:

$$R_0 = \text{o}''\cdot527, \qquad R_2 = R_4 = -R_0, \qquad R_6 = \infty, t_1 = \text{o}''\cdot075,$$
$$t_3 \text{ (cement)} = \text{o}\cdot000, \qquad t_5 = \text{o}''\cdot025.$$

Diameter = o″·350. Clear aperture = o″·300. Two such achromatic doublets placed with crown elements facing each other form the erector. The distance from vertex to vertex for coincidence of principal planes = o″·415, but the distance I prefer is only o″·050. In this latter case the inner principal planes of the separate doublets coincide (Fig. 2). The achromatism of the telescope may be improved and

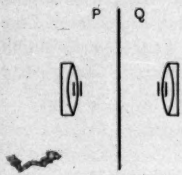

Fig. 1.

Fig. 2.

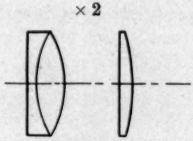

Fig. 3.

the construction further simplified by substituting for the second doublet a single lens made from a quite ordinary crown glass (Parra-Mantois No. 6132, type 3970) with radii of curvature $R_0 = 2''\cdot510$, $R_2 = -\text{o}''\cdot834$ (Fig. 3). The proper distance apart is best found by experiment. I prefer this combination to any other. It is an excellent compromise between an extensive flat field and best achromatism*.

The Huygenian part of the erecting eyepiece was made of Schott's boro-silicate crown, type O. 2188, and was constructed by Mr C. Baker.

The data are: Field lens, radii 1″·128 and ∞, diameter 1″·150; Eye lens, radii o″·518 and ∞, diameter o″·500; Lens separation, vertex to vertex, 1″·964; Focal length 2″·00. But if field is no object and the very best achromatism obtainable from a doublet telescope is necessary, then the simpler Dollond non-achromatic erecting eyepiece is best.

Now returning to the O.G. and then to the telescope generally; in the former, by adopting the one radius construction, the outstanding colour has been changed over from o·002176 undercorrected to − o″·005718 overcorrected. The spherical aberration from o″·005096 has become o″·000680. Finally the difference of real focus for ray Λ axis and edge has been reduced from o″·034428 to o″·030681. With regard to the latter, the relative aperture at first attempted was $f/5$, but marginal astigmatism was somewhat prominent. By reducing the aperture to $f/7$, Mr Nelson, who very kindly helped me with the critical work, got such an improvement that it was decided to recompute the doublet on the basis of $f/7$ with a power variable at will from 25 to 35 diameters. This was ready in April 1916 and was duly sent to the Ministry of Munitions and afterwards to the War Office.

* The position of the erector in the telescope depends entirely on the magnification required.

At this point, unfortunately, the only existing example of the little *f*/7 which had been sent to the War Office was unaccountably lost, with the result that complete new drawings had to be made and a new mount designed. Thus it was not until 24th Jan., 1917 that another, the first of a dozen offered to and accepted by the War Office, could again be sent in. At last, after many delays, a portion of these were sent out for trial to France. But happily trench warfare was practically at an end and the demand for a short high power telescope was no longer felt. In order therefore that the little *f*/7 may not be lost altogether, I have been asked by Professor Cheshire to write a description of it, and the present short paper is the result.

Note added 14th May, 1920. In calculating the short-focus one-radius objective described above, line A, of wave-length 560·7$\mu\mu$, was used as principal ray. It is to this fact that I am inclined to attribute the excellence of its performance. Line A lies approximately midway between *D* and the mercury line of wave-length 546·1$\mu\mu$ used by Continental computers (see "A Revised System of Comparative Optics," *The Optician*, 2 July, 1915).

DISCUSSION

Professor Cheshire explained the circumstances under which, during the war, the demand for a short high power telescope arose. When he was at the Ministry of Munitions they had repeated demands from officers at the front for a short high power telescope which could be used for observation purposes in a restricted space. All available telescopes of this type were examined, both at the Ministry and at Woolwich, and found to be useless. They gave a high magnification it is true, but the definition was sadly lacking and the light-collecting power was of necessity very small. It was clear that, as in so many other cases, the men who had used and been impressed with the power of the then available short high power telescopes had not examined them critically; they had obviously failed to notice that the higher magnifications attained were not accompanied by a corresponding increase in the defining power. Under these circumstances he had written to Colonel Gifford, asking him if he would take the problem in hand. Colonel Gifford took up the matter with great enthusiasm and replied that he thought he would be able to solve the problem with optical glass which he himself possessed, and which he would be glad to place at the disposal of the nation for the production of the telescope in question. He thought that the results of Colonel Gifford's investigation should be placed on record, so he had requested him to communicate them to the Society.

Instructor-Commander T. Y. Baker said that the telescope which had been described was just the instrument for which he was looking. In connection with many of the instruments with which he had to deal he had realised the necessity of having high power telescopes of reasonable size.

Mr Conrad Beck said that his firm had designed a new form of small high power telescope, with an apparent angle of 40°, for sketching purposes, where the observation post was a confined space such as to render the use of large telescopes

impossible. The objective had a focal length of about 11 inches and an aperture of ½ inches. The erection was obtained by means of Porro prisms. A positive lens and a negative lens were placed between the objective and the eyepiece, in the focal plane of both, so that they were inoperative in ordinary circumstances, but when separated caused an increase in magnification. It was essential to use a double lens, otherwise, on displacing the negative lens, the eye-point would be in the wrong position. The focussing motion was at the objective end, and the magnification could be altered from 15 to 70 by extending the eyepiece.

Mr P. F. Everitt asked what the apparent field of view was. He thought it appeared fairly small.

Mr R. S. Whipple remarked that it would be a loss to the community if an instrument which appeared to have so many good technical qualities should not be employed, now that it is no longer required for war purposes. He suggested that it might be of service in deer-stalking, yachting, etc.

The author, in reply, said he wished to thank Prof. Cheshire and Col. Williams for the great assistance he had received from them in connection with the investigation.

He was interested to hear the description of Mr Beck's instrument. The employment of a negative lens in conjunction with the eyepiece was most interesting, and he would certainly try the system.

He was unable to state exactly what the apparent field of view was, but of course it varied somewhat with the type of instrument.

The late Sir David Gill had made use of one of his short telescopes for deer-stalking.

THE No. 7 DIAL SIGHT, MK. II

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THE No. 7 Dial Sight was an instrument required in very large numbers during the war for artillery guns of nearly all calibres. Most of the instruments found in England before the war had been obtained from Germany, so that very little was known of the details and difficulties of manufacture. In experience it has been found that the adjusting and testing of this dial sight is of a very exacting nature, and manufacturers had to evolve for their own use necessary and sufficient methods for the purpose. I have had considerable experience with this instrument and have therefore written this paper regarding some of the methods of adjusting and testing employed.

The instrument itself (Fig. 1) may be looked upon as a periscopic theodolite, having a revolving head and fixed eyepiece, by means of which angles in azimuth can be determined to about 3 minutes of arc. The dial plate is divided in degrees,

Fig. 1.

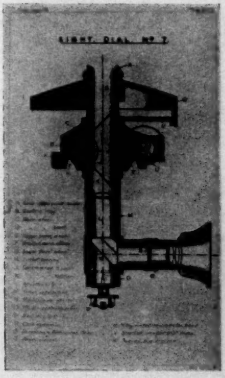


Fig. 2.

and the micrometer drums into 10 minute divisions, one complete revolution of the latter giving 5 degrees azimuth deflection. The maximum allowable error was $\frac{1}{3}$ of a drum division, i.e. about 3 minutes.

In use, a convenient and fixed horizontal direction is first sighted by means of the dial sight, and the vertical plane containing the axis of the gun adjusted with respect to this direction until it passes through the target, which may or may not

be visible. Thus the dial sight allows of a gun being fired accurately in a predetermined direction by the simple expedient of sighting any well defined object in any arbitrary direction in a horizontal plane. A gun, for example, may be trained on an object by sighting a mark, such as a church steeple to the rear of the gun.

A section of the instrument (Fig. 2) shows the interior of the sight, and it will be seen that the dial-plate *W* and the upper prism holder *A* are orientated by the worm-wheel *C* driven by a worm-spindle engaging in the teeth *D*. At each end of the worm-spindle are attached the micrometer drums *U*.

As it is at once evident that a considerable part of the accuracy of the instrument depends on the worm and worm-wheel, it is necessary that these should be given a very stringent test before the instrument is assembled in any way.

Test of Eccentricity of Worm-wheel. The body of the sight *S* (Fig. 3) carrying the worm and worm-wheel is mounted rigidly in a suitable stand *H*, and levelled.

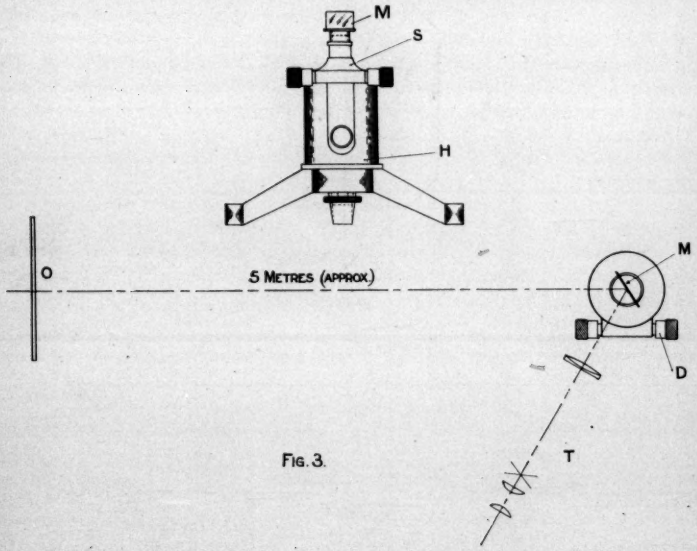


FIG. 3.

A parallel mirror *M* ($\frac{1}{4}$" thick) fully silvered on both surfaces is placed centrally in a jig which fits in the top of the projecting worm-wheel. At a distance of 5 metres (approximately, but not less) from the centre of rotation, and in the same horizontal plane as the centre of the mirror, is fixed a piece of white celluloid *O* with a thin line engraved on it (white celluloid is used as very fine black lines can be engraved on it, and it is suitable in every way for optical purposes). By placing a telescope *T*

in the position shown, the line on the celluloid may be viewed by reflection from the mirror *M* and made to coincide with the cross-webs in the focal plane of the

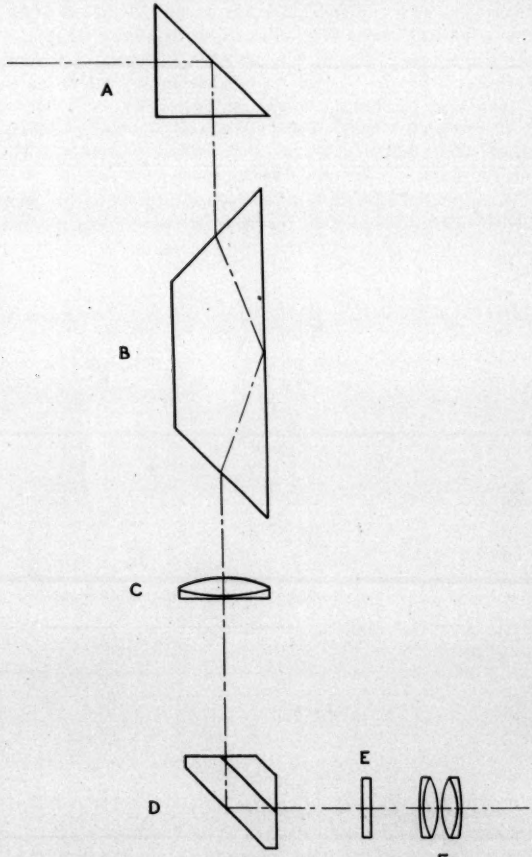


FIG. 4.

eyepiece of the telescope. If the micrometer drums *D* are already at their zero-reading, it is at once evident that, by the rotation of these, the worm-wheel may be

turned through 180° (carrying the mirror with it, of course) until the image of the engraved line is brought into the telescope by reflection from the *second* surface of the mirror *M*. Now, when the image by the second reflection is brought on to the cross-webs, the micrometer drums can again be read; and in more cases than not, it will be found that the readings are a few minutes "high" or a few minutes "low" with respect to the "true" 180° reading. This error is brought about by eccentricity of the pitch-circle of the teeth of the worm-wheel cut by the hobbing machine. So that by taking a series of 180° readings in different positions (say, every 45°) round the worm-wheel and noting the results, a very definite idea may be obtained as to the position of the diameter on which the greatest eccentricity lies. If the error by the drum readings exceeds 3 minutes, this is not sufficiently accurate, and by a very careful process of grinding the teeth of the worm-wheel in the correct position this may be corrected; 3 minutes error represents ·0041 inch eccentricity at the centre. Thus it will be seen that quite a high accuracy may be obtained by this method, which proved quite satisfactory.

Centre Prism Adjustments. Fig. 4 shows the optical system (in side elevation). The lower portion, marked *C*, *D*, *E*, and *F*, forms a simple terrestrial telescope of magnification 4 diameters. *E* is a graticule with a cross-line on it in the focal plane of the eyepiece *F*. *A* is a right-angled prism which rotates with the dial plate. The prism *B* serves to keep the field of view always erect whatever orientation is given to the upper prism *A*. In order to effect this it is necessary that the prism *B* shall rotate at just *half* the speed of the upper prism *A*, this being done by means of a bevel drive on the inside of the worm-wheel.

It is of the greatest importance that this automatically erecting centre prism should be made and adjusted in position, so that the emergent axial ray may be collinear with the incident axial ray in any position of the prism, to within 1 minute of arc. To attain this accuracy the following method was employed. A collimator

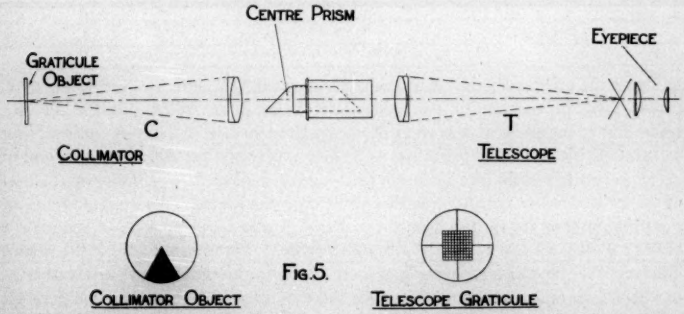


FIG. 5.

C (Fig. 5), was used (focal length of O.G. = 11 inches) which had as an object a graticule of the design shown, a dark sector of 60° with its apex at the centre.

Mounted on the same rigid base as the collimator is a telescope *T* (focal length of O.G. = 11 inches) which is so aligned that the intersection of the two main cross-lines of a graticule in the telescope coincides with the image of the apex of the object. The telescope graticule is also shown, the small squares representing an angular subtense of 2 minutes (0·14 mm.). The fitting in which the centre prism mount rotates is held between the collimator and telescope in such a position that its axis is collinear with their common axis. It is then evident that, if the prism mount is revolved, the dark sector of the object will also appear to revolve when looking through the telescope, at the same time keeping its apex at or near the centre of the graticule of the telescope. When correct adjustment of the prism is attained, no movement of the image of the apex will be noticed. Besides the accurate setting of this prism it is very essential that it shall be held extremely firmly in its mount, owing to the rough usage such a sight has to stand. One would at once be tempted

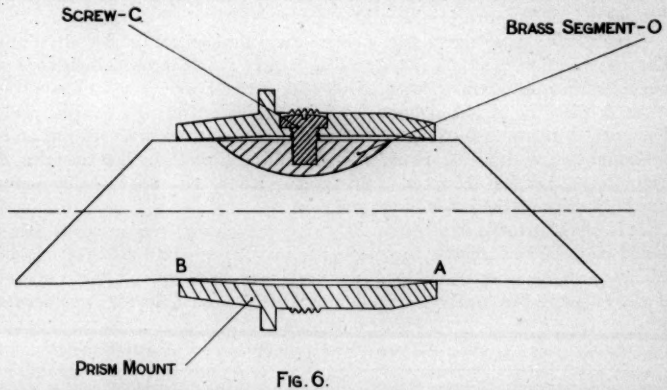
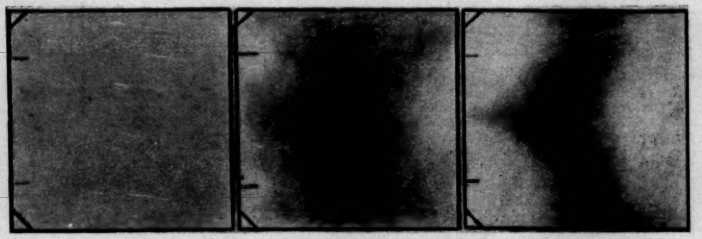


FIG. 6.

to clamp the prism very tightly or perhaps even cement it; either of these, however, proves fatal, for any undue strain imposed on the prism will produce a double-refracting effect and the definition of the instrument will suffer very seriously on account of this. Thus the prism has to be held firmly and yet without strain, and in order to test for strain a polarisation method was employed. Fig. 6 shows a section of the prism mount with the method of holding the prism. Two metal tongues of the prism mount are sprung inwards at *A* and *B*. An opposing screw *C* bears on a shallow metal segment *O* which is let into a groove ground in the side of the prism. The screw *C* when shouldering home on the prism mount has to exert sufficient pressure to clamp the prism. These are the only means of support; usually small strips of paper are put at *A* and *B* to prevent scratching of the glass. Now it is evident that, as the points of support of the prism at *A* and *B* are a considerable distance apart (1½ inches), quite a small excess of pressure with the screw *C* will

have a considerable tendency to "sag" the prism in the centre, and in order to get an idea of the strain imposed, two Nicol prisms are placed one either side of the prism and are crossed, so that on viewing through the second Nicol strain will be shown up as light patches. Fig. 7 shows the kinds of appearances that are obtained;



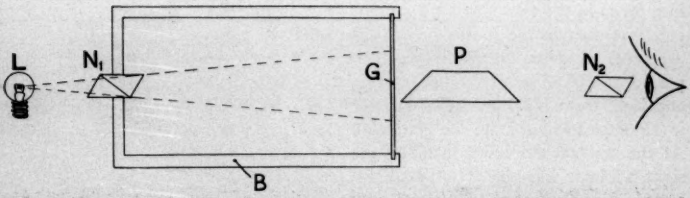
(*a*) (*c*) (*b*)

PRISM VIEWED END ON BETWEEN 'CROSSED' NICOLS

Fig. 7.

(*a*) insufficient pressure, (*b*) too much pressure, (*c*) the appearance when the prism is clamped just as tight as possible without affecting the definition (this is found by experiment of course). I mentioned just now that a Nicol was placed in front of the centre prism whilst testing; this is scarcely correct, as Nicol prisms of such an aperture are rather expensive and not very easy to get. So I employed a method by which quite a small Nicol could be used and yet the entire field of the centre prism be filled with polarised light. The method is shown in Fig. 8. An en-

FIG. 8.



closed box *B* had at one end a Nicol *N*, and at the other a piece of ground glass *G* (3 inches × 3 inches). A lamp was placed at *L*, so that the ground glass was entirely illuminated. The prism *P* in its mount was then placed as shown and viewed with a second Nicol N_2 in front of and close to the eye. (N.B. The ground glass screen has no depolarising effect as might be expected.)

The next adjustment of the sight is the focussing of the telescope; this was carried out by an auto-collimation method. The diagram (Fig. 9) illustrates the portion of the sight in which the telescope is contained. A suitable stand was so arranged that this part of the sight and a telescope *T* (focal length of O.G. = 11¼ inches) were rigidly fixed on the same base, and also that the former could be quickly

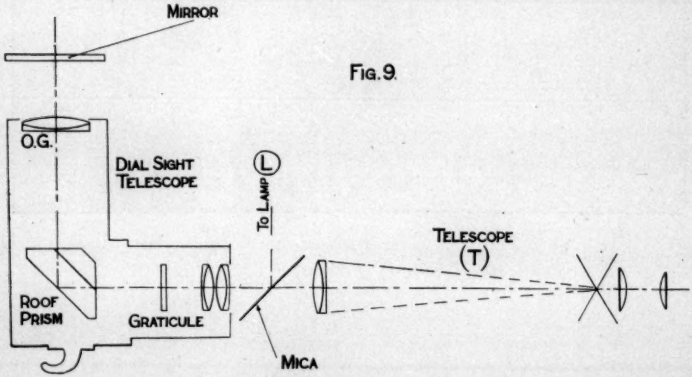


FIG. 9.

taken in and out of position. The first part of this adjustment consisted of setting the graticule in the focal plane of the eyepiece. To do this the cross lines of the graticule were viewed by the telescope *T* (which was already set for parallel light) through the eyepiece combination, and the distance between graticule and eyepiece adjusted until brought sharply into focus in the telescope *T*; the graticule was then in the focal plane of the eyepiece. A piece of mica was then placed at 45° in front of the eyepiece and light from a lamp *L* reflected into it (in order to illuminate the graticule). At the top of the lower half of the sight a mirror was placed so that light, after leaving the O.G., would travel back along its original path and come to a focus again, thus giving a back-reflected image of the graticule cross-wires at or near the *real* cross-wires. Then, observing through the telescope *T* and by adjusting the position of the dial sight

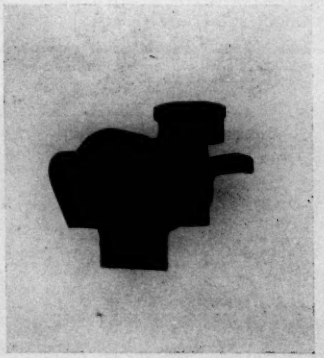


Fig. 10.

O.G., the image can be brought to focus simultaneously with the real cross-wires, thus ensuring that the graticule is at the focus of the O.G. This method is extremely good, as very small errors in focussing may be detected.

Fig. 10 shows the upper prism holder or the head of the sight, and although there is scarcely any adjustment for this part, it is interesting to note the method of holding the right-angled prism firmly in position. Fig. 11 shows the various

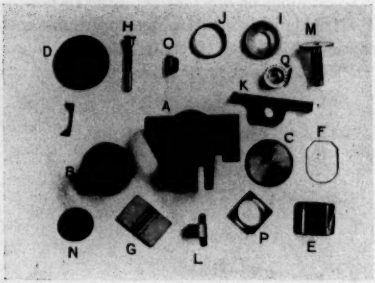


Fig. 11.

A = Upper prism holder case.
B = ,, ,, mount.
C = ,, ,, mount cap.
D = ,, ,, holder cap.
E = Right angled prism.
F = Glass protecting window.
G = Aluminium plate.
H = Worm spindle.
I = Milled head.
J = ,, ,, drum.
K = Open sight.
L = Steel spring.
M = Worm spindle bearing.
N = ,, ,, head.
O = Segment intermeshing with worm spindle.
P = Prism mount segment.
Q = Open sight gear wheel.

components that go to make up the mount for the prism. The prism rests between two metal segments (one of which is moveable), each having four small square projecting feet. The aluminium plate rests on the hypotenuse surface and the spring, exerting pressure on this all the time, keeps the prism firmly in place. This upper prism holder has an open sight on it. It is of the usual rifle-sight pattern, i.e. the inverted V at the front and the ordinary V at the rear of the sight, and is elevated

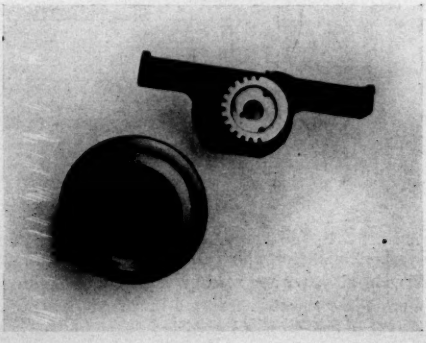


Fig. 12.

or depressed with the prism by means of the milled head. Now, as the prism acts as a mirror, any vertical angular movement is just doubled by reflection (as is well known), so, in order that the open sight shall always be directed at the same object as is seen on viewing through the eyepiece of the instrument, it is necessary that the open sight should travel at just double the speed of the prism. This is done by a 2 to 1 gearing rather cleverly and compactly concealed at one end of the outer case of the upper prism holder. Fig. 12 shows this end with teeth cut in it over a part of the circumference, also the teeth on the open sight which mesh into it.

Testing a complete sight. The sight was held firmly in an inverted position directly over the centre of a spectrometer table, which was specially selected for good dividing, being almost entirely free from eccentriticy error (Fig. 13). The spectro-

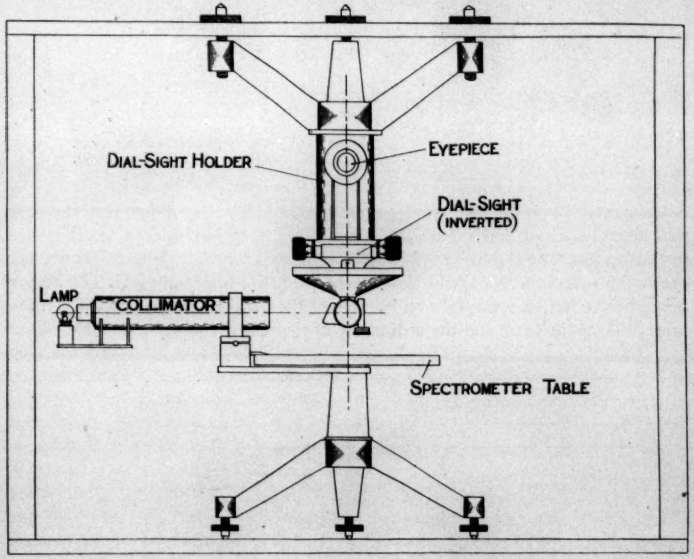


Fig. 13.

meter had a moveable collimator which could be rotated round the entire circle (which was 9 inches in diameter and read by vernier to 0·5 minute). At the focus of the O.G. was a cross-line object illuminated by a small lamp carried round with the collimator. It is then evident that by placing the collimator opposite the upper prism holder and looking in at the eyepiece of the dial sight, the cross-line object

will be seen; and by bringing the divided drums of the sight into action the object may be brought on to the cross-line on the graticule of the sight. Thus, by moving the collimator into different positions round the circle (say every 30°) and setting it by means of the vernier, and then following the object with the dial sight, readings given by the sight may be obtained and the instrument tested in this way. The method of ensuring that the axis of the sight was directly above the centre of the spectrometer table, was to place in the sight holder the lower half of the dial sight (this formed the telescope) which was focussed on an extremely small hole made purposely at the centre of the table when the circle was in the process of being turned in the lathe. Both the spectrometer table and dial sight holder were levelled by levelling screws (as shown) and when this small hole was laid on the intersection of the cross-lines of the dial sight telescope, it is quite evident that these two were directly in line.

DISCUSSION

Mr Conrad Beck said that the statement that before the war all the dial sights were made in Germany was not accurate, nor was it fair to the Army authorities. The majority of those used in the British Army were made in England. This was due to the energy of the Woolwich inspectors and their co-operation with manufacturers. His firm had been asked to make dial sights several years before the war, and the results were at first bad, because they had no idea of the great difficulties involved until they actually manufactured the instruments. One of these difficulties was the great skill required in making the revolving fittings for prisms. In the centre prism only one of the errors can be eliminated by adjusting the prism; the others must be corrected in the construction of the prism. The angle between the faces of the roof prism must be accurate to within about 3 seconds. Several of the mechanical parts are extraordinarily difficult to make. Three parallel fittings, two of which are in one piece and one in another, must be retained on one axis, although the two pieces must be taken apart and put together many times. Great patience and care in making the necessary tools were requisite.

Lt.-Col. A. C. Williams said that dial sights were instruments which must be made in mass production. It would be impossible to get a single firm to tender for only a few instruments.

During the war a number of difficulties had been experienced in connection with dial sights. The inside parts had at first been made of steel, with the result that they ultimately seized. When phosphor bronze and steel had been used, galvanic action took place and this started corrosion. He thought that the Germans employed the same metal throughout, some of the parts being made of soft steel and others of hardened steel.

The question of lubrication was important. The mineral jelly or vaseline used as a lubricant must be absolutely pure and free from volatile constituents, otherwise deposits would form on the glass surfaces, especially in instruments used in hot climates.

The question of eliminating moisture from the instruments when being assembled and of preventing its entrance into the completed instruments was also very important.

He thought that the method employed for testing strain in the centre prism would not be very satisfactory as the ground glass would destroy the polarisation of the beam of light. At Woolwich he had employed as polarisers heliograph mirrors, painted black on the under surfaces, and he thought that such an arrangement was better than a pile of plates.

Lt.-Col. J. W. Gifford asked about the weathering qualities of the glass used for the prisms. He had found that a boro-silicate crown with a ν value of about 60 is very apt to sweat, while barium crowns tend to effloresce, so that one is almost driven to employ a light glass such as Schott's prism glass.

Mr Hanson said that at Woolwich the chief difficulty in connection with filming was found with the graticules. The whole question of filming was one that still required a great deal of experimental investigation. Most of the methods of test employed by the author were different from those used at Woolwich.

Dr J. S. Anderson said that in the strain test the depolarising effect of the ground glass was not very serious—a large percentage of the light remained polarised. The test, as employed at the National Physical Laboratory, was somewhat more sensitive, as a wave plate was used for giving the sensitive violet tint. Even without the wave plate, however, the method was quite satisfactory for giving comparative results with regard to strain in large pieces of glass.

Professor Cheshire drew attention to the beautiful specimens of dial sights, one entirely dissected, which had been presented to his Department by Mr Conrad Beck. He suggested that the high-water mark reached by others in the manufacture of optical instruments during the war would best be kept on record by their following Mr Beck's generous example.

In the early days of the war he and his colleagues at the Ministry of Munitions had been greatly worried by the insistent demand for dial sights. After a great deal of arduous work Messrs Beck and Messrs Ross had been able to supply the instruments in great numbers, so that towards the end of the war the problem was no longer a source of worry.

Mr Chapman said that between 1909 and 1911 Messrs Beck first attempted to manufacture dial sights. The first instrument was sent to Woolwich about 40 times before it was passed. He wished to express his thanks to the Woolwich inspectors for their great patience in these early days.

He thought that Mr Johnson had referred only to a few of his troubles. The difficulty of mounting the centre prism was brought home to him in 1909, when he found that in one instrument astigmatism was introduced owing to the prism being held so tightly that its hypotenuse was converted into the surface of a cylinder.

For the purpose of assembling dial sights Messrs Beck had eventually made use of 48 different pieces of apparatus for separate tests. A great deal of the delicate work of assembling had been carried out by girls under the supervision of a skilled examiner.

THE CONTACT SETTING OF SMALL DISCS

By A. BARBARA DALE

(Optics Department, National Physical Laboratory.)

MS. received, 25th May, 1920. *Read and discussed, 10th June*, 1920.

The investigation described in this paper was the outcome of certain points raised in Part II of Mr French's paper on "The Unaided Eye*." Mr French there gave the values obtained for the apparent diameters of small bright circular discs against a dark background, the diameters being measured by setting the discs in tangential contact. For the purpose he used the two images of a circular object produced in the ordinary way by a double image prism, the circles being set in and out of contact by a longitudinal motion of the prism. This method has obvious defects in practical use, and an alternative method of using double image prisms, in which no translation of any part of the apparatus is necessary, was suggested to me by Mr Guild.

Two double image prisms, each consisting of one glass and one calcite component, were mounted one behind the other in a cylinder so that their relative orientation could be altered at will, and the cylinder was placed on the table of a spectrometer. The object, a bright disc on a dark ground, or *vice versa*, was placed about 5 metres away, and, when the prisms were suitably adjusted, images were produced which by rotation of the table could be brought into coincidence or separated to any desired extent in a horizontal direction.

In general, the two prisms mounted in series produce four images of an object, the front prism forming two polarised images each of which is duplicated by the second prism. When the prisms are so orientated that the crystallographic axes of their calcite components are parallel two images only are formed. Figs. 1 and 2 show diagrammatically the paths of rays through the system under these conditions, the direction of the crystal axes being indicated. In the arrangement of Fig. 1, in which the cemented interfaces are parallel, the two prisms produce double the deviation of one, and two widely separated images are seen. In the arrangement of Fig. 2 the second prism produces a deviation opposite to that of the first, provided that the prisms are similar and that the incident ray is in the direction shown, and this was the arrangement adopted in the investigation. Since the angular separation produced by a double image prism varies with the angle of incidence of the rays on the cemented interface, a rotation of the table on which the prisms rest will increase the separation due to one of the prisms, and diminish that due to the other. Hence, on looking at a distant object through such a system, two images will be seen which may be made to coincide or to separate in either direction by varying the azimuth of the prisms.

* *Trans. Opt. Soc.* 21 (1920), 1.

The two prisms used were not rectangular, the outer surface of the glass component being inclined at an angle of about 5° to that of the rectangular calcite component. The introduction of this prismatic angle causes a ray falling normally on the calcite component to produce two images equally deviated to either side of the normal.

The tracing of rays through two prisms in the position of Fig. 2 shows that for all angles of incidence less than 15°, measured from the zero position, the deviation between the two emergent rays is very approximately directly proportional to the angle of incidence. As the angle of incidence increases above this value the rate of increase of the deviation becomes greater.

It is evident that the apparent separation of the two images of a distant point is proportional to the angle which the axis of the prism system makes with its zero position, that is, the position giving coincidence of the images. With the prisms

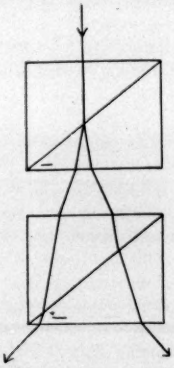


Fig. 1.

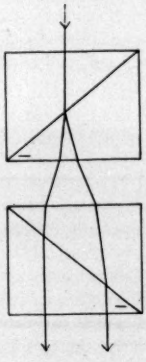


Fig. 2.

used a rotation of 2° in either direction produced an apparent angular displacement of 9′·3. The error in measured separation is therefore only about ·08 of the error in measuring the rotation of the prisms, so that a highly accurate circle is not essential. The arrangement is calibrated on an object of known dimensions large enough to render the errors of observation negligible, but not so large as to fall outside the limits of direct proportionality between the angle of obliquity and the separation of the images.

This method of producing a double image has one distinct disadvantage. Both images are slightly deformed, the one being elongated and the other compressed along the direction of displacement. It will be seen at once that this deformation is consequent on the property of the system by which oblique rays are deflected from their normal direction on emergence. Suppose that a rotation of the prisms by 1 minute of arc from the direction of zero separation produces a separation of

α minutes between the images of a point. In Fig. 3 let AB represent an object of angular diameter d, the centre of which is displaced from the zero axis OP by an angle θ. The two images a_1 and a_2 of the edge A will be separated by an amount $\alpha\left(\theta - \frac{d}{2}\right)$, while the angular separation of the two images b_1 and b_2 of B will be $\alpha\left(\theta + \frac{d}{2}\right)$. An observer looking at the object through the prisms at P will therefore see two images, a_1b_1 and a_2b_2. Clearly, the angular width of a_1b_1

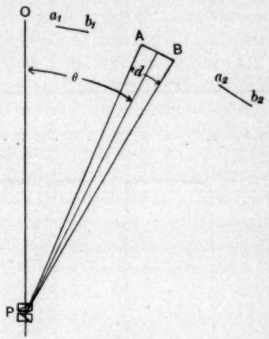


Fig. 3.

$$= \angle a_1PA + \angle APB - \angle b_1PB$$
$$= \tfrac{1}{2}\alpha\left(\theta - \frac{d}{2}\right) + d - \tfrac{1}{2}\alpha\left(\theta + \frac{d}{2}\right)$$
$$= d\left(1 - \frac{\alpha}{2}\right).$$

Similarly, the angular width of a_2b_2

$$= d\left(1 + \frac{\alpha}{2}\right).$$

That is to say, the deformation of each image is $\frac{\alpha}{2}d$ in the direction of the deflection, one image being compressed to the same extent that the other is elongated, and this deformation is independent of the distance of the object from the axis, provided that the object is small.

In taking measurements of the apparent diameter of a circle by the method of setting two similar circles in contact, first at one edge and then at the other, it is obvious that the deformation of one image counterbalances that of the other, for the shift between the two positions is equal to the sum of their diameters, which is the same as twice the diameter of the normal image. No correction has to be applied, therefore, on account of this effect. In the actual investigation the value of the quantity α which governs the deformation was $0'{\cdot}155$, that is to say, the horizontal diameters of the images were about $\frac{1}{18}$ and $\frac{1}{13}$ respectively of the vertical diameter. Although this was recognisable in the case of the larger circles, it was not detectable with the smaller circles, and could have no appreciable effect on the nature of the settings nor on the results obtained.

Two separate series of experiments were carried out, the first dealing with bright circles on a dark background and the second with dark circles on a bright background. In each series the size of the object-was varied from one subtending an angle of $0'{\cdot}8$ at the prism to one subtending an angle of $11'{\cdot}1$, and measurements were taken of the apparent diameter with and without the use of a small diaphragm in front of the eye.

The curves to the right in Fig. 4 show the results obtained in the case of bright circles, the illumination being just below that producing the star appearance in

the image. The ordinates of the curve represent the true diameter, and the abscissae the apparent diameter of the object. If there were no error of adjustment the ideal curve would be the straight line *A* at 45°. The actual curves show deviations from this ideal line, and in general character are very similar to those obtained by Mr French. At full aperture (curve *B*) the eye registers too small a diameter for objects between 2′ and 9′, the error not exceeding 15″. For larger objects the tendency seems to be to record too large a diameter. When a diaphragm of 3·8 mm. was used just in front of the eye (curve *C*), there was a fairly constant over-estima-

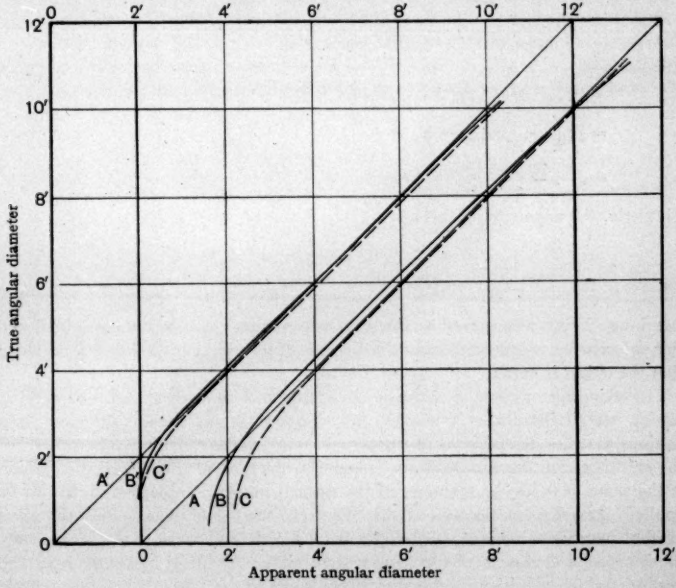


Fig. 4.

tion of the diameter, but this was very slight indeed, and for circles larger than 4′ the error was small in comparison with the accuracy of repetition of the settings. For very small objects there is an unmistakable increase in the error; with the full aperture of the eye the curve bends away from the ideal curve and tends to a minimum value of about 1′·5 for the apparent diameter of all objects less than 1′ in size. Using the diaphragm, the limiting value is in the neighbourhood of 2′.

The fundamental characteristics of these curves agree closely with those of the curves obtained by Mr French, and the differences in detail are on the whole no greater than could be accounted for by different experimental conditions and by

the fact that the series of settings described in this paper were made by one observer only and would therefore be more greatly affected by errors due to the personal element.

The second series of curves to the left in Fig. 4 relate to dark circles seen on a bright background. It was found impossible in this case to make settings with circles as small as 1′ in angular diameter, not because a black object of this size was invisible, but because the bright background of one image fell over the dark object of the other, diminishing the contrast of dark and light too greatly to make measurements possible. The nature of the curves for small circles as far as that point is, however, very similar to that of the curves for bright circles. At full aperture (curve *B′*) the eye appears to make very little error in its estimation of the contact position for objects greater than 2½′, although a positive and increasing error appears in the settings of circles over 8′. As before the curve tends to a limiting value of about 2′ for very small circles. When a diaphragm is used (curve *C′*) the error is greater; there is a fairly constant over-estimation of the diameter by about 15″, until the object used is smaller than 4′. The curve then tends to a limiting value of rather over 2′.

The description of the curves is much simpler than an adequate explanation of their characteristics. There are three main facts on which any theory must be based: first, the similarity of the curves for bright objects and for dark; second, the nearly constant minimum apparent diameter of 2′ reached for all very small discs, and, third, the marked diminution of the error as the size of the discs is increased above 3′. With regard to the second point, Mr French* suggested that the light insulation of the sensitive elements of the foveola might be deficient, so that the influence of a bundle of rays, however small, could not be restricted to one element alone. This explanation might suffice in the case of the bright discs, but cannot explain the similar results obtained in the case of the black discs.

It has also been suggested that there are two effects contending for supremacy, the effect of diffraction and the effect produced by an unconscious tendency on the part of the observer to produce overlap of the circles rather than contact. This latter effect would presumably act in the direction of reducing the apparent size of the image for objects of all sizes, both dark and light. The effect of diffraction is not so uniform. I am indebted to Mr T. Smith for a suggestion that a possible explanation of the different phenomena might be obtained from a study of the actual diffraction effects under different conditions, and there appears to be strong support for attributing the main characteristics of the curves to these effects.

Consider first the diffraction image of a very narrow slit. The full line in Fig. 5 shows the well-known curve representing the intensity of the image which by a geometrical construction would be represented by the line of *AB*. A second intensity curve in broken line represents the diffraction image of a second slit *CD*. The intensity curve for a wide slit is obtained by integrating this curve over a horizontal distance equivalent to the width of the slit, and the result for an infinitely wide slit, or a uniform field terminating in a straight edge, gives a curve sloping up

* *Trans. Opt. Soc.* 21 (1919–20), 20.

from a minimum intensity at a point outside the edge of the geometrical image to a maximum at a point inside the edge (Fig. 6). In the contact setting of two images it is not immediately obvious what line the eye would select as the apparent edge of either image, but it appears probable that it would choose as the contact position that configuration of the images which gave no change of intensity along their common diametral line. Fig. 6 shows the intensity curves for two similar images so placed relatively to each other that the sum of their intensities at any point is equivalent to the maximum intensity of either image. Since a reversal of light and dark about the actual edge of the object must produce a corresponding reversal of light and dark in the diffraction image, this image must be symmetrical about the edge *EF* of the geometrical image. Hence the intensity curve must cut this edge at a point where the intensity is one half of the maximum intensity, and, to satisfy the above criterion of the contact setting of two similar images, their intensity curves must cross each other on the edge of the geometrical image. With a very wide slit, therefore, the contact setting should show little or no error due to diffraction.

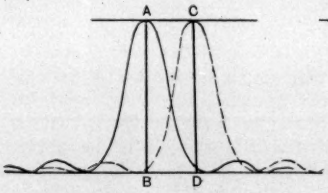


Fig. 5.

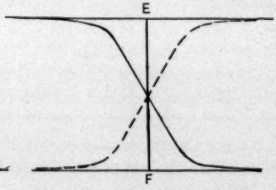


Fig. 6.

Using the same criterion in the case of small objects, there will obviously be an appreciable error in the apparent contact position, and, as the width of the object diminishes, this will assume the minimum value represented by *AC* (Fig. 5), where the slits are so placed that there is no change in the intensity of illumination between them. This position will be slightly closer than that fixing the resolving distance between two narrow slits, so that the minimum error in apparent diameter will be slightly less than the resolving distance. Moreover, these effects apply equally to bright and to dark objects, and will be increased as the limiting aperture of the system diminishes, explaining the larger error shown in curves *C* and *C'*.

Although perhaps some small modification should be made in applying these results to the case of discs instead of slits, the necessary change should be slight and should not alter materially the relation of the effects called into play under the various circumstances. If this be so, the theory is well borne out by the facts as they are recorded in the curves; the probable additional effect due to the tendency to overdo the contact setting may well explain the negative error in the apparent diameter shown in curves *B* and *B'*.

In conclusion, I should like to express my warm thanks to Mr Guild, at whose

suggestion this investigation was undertaken, for his ever-ready help and criticism throughout the work, and to Mr Smith for his illuminating suggestion as to the interpretation of the experimental results obtained.

DISCUSSION

Mr T. Smith said that some time ago he had considered, from the point of view of diffraction theory, the problem of the relation between the apparent separation of two slits and their actual separation. The curve connecting these quantities for different widths of slit is very similar to the curves which Miss Dale had obtained in the case of discs. It periodically crosses and recrosses the 45° straight line which represents the case of no error in setting. If it were found that the problem of circular discs, when worked out on the same basis, gave similar results and the crossing points agreed with experiment, it would follow that all Mr French's results could be explained by means of diffraction phenomena.

Instructor-Commander T. Y. Baker asked if the measurements were made from separation to contact or from overlap to contact. He had found that with bright discs on black backgrounds the latter type of setting gave better results than the former; the discs employed subtended an angle of about 1°.

Mr P. F. Everitt enquired if the order in which the settings were made had been varied. It had been shown that the variation of error found in the visual bisection of a line of given length could be traced through a reasonable number of settings.

Mr L. C. Martin asked if measurements had been made of the luminosity of the background and discs.

Mr J. Guild said that Miss Dale's results, by demonstrating that the phenomena of contact setting of circular areas was the same for black circles on a white ground as for white circles on a black ground, disposed of any theory depending on the imperfect light insulation of the retina. Not only so, but the theory advanced for the phenomena gives simple explanations of many of the minor peculiarities. For instance, in discussing Mr French's paper, he (the speaker) had suggested as an explanation of the effect of a diaphragm on the errors of setting with the larger circles that two opposite effects were in operation, viz. a tendency on the part of the observer to overshoot the setting, thereby making the measured diameter too small, and an effect depending on the aperture of the eye, and probably therefore due to diffraction, which made the areas of the retinal images too large. The relative magnitudes of these effects would determine whether the actual measurements were too small or too large.

It is suggested in this paper that the setting adopted is that in which the diffraction patterns at the edges which are in contact are so overlapped that the resultant intensity of illumination is uniform along a line passing through the point of contact. In the case of circles it at once becomes evident how a tendency to overshoot this setting arises. The eye integrates brightness over a small area surrounding the point of contact; consequently, in order that the average intensity over this area may be equal to that inside the circles, a condition which is necessary to prevent

an apparent dip in the intensity curve where it crosses the contact, the intensity at the actual line joining the centres must be a little too high in order to balance the reduced intensity to either side of it where the edges are receding. Clearly this effect will diminish as the curvature of the circles diminishes and should be zero with rectangular areas. This fits exactly with the behaviour, for larger radii, of Miss Dale's curves. The explanation of the contrary tendency is not at once apparent, since on the theory, for circles large enough for the diffraction pattern to be symmetrical with respect to the edge of the geometrical image, the setting (apart from the overshooting tendency) would give no error. It may be, however, that with the size of circles employed this stage had not been reached, and that the mid point of the diffraction curve is still outside the geometrical edge, in which case all the phenomena of the experiments are explained.

The author, in reply to Commander Baker, said that all the readings had been made from separation to contact; those made from overlap to contact were found to be very variable.

In answer to Mr Everitt, she stated that the order in which the measurements were made was generally from large discs to small ones, but occasionally some of the settings were made out of order. The maximum variations found amounted to about 12 seconds in the case of the bright discs and a rather larger figure for the dark discs. She had found that the first set of observations made at the beginning of the day had always to be discarded on account of their great variation.

In reply to Mr Martin, she said that no photometric measurements had been taken of the illumination; in the case of the bright discs the intensity had been put below that which produced a star appearance of the disc; with the dark discs a fair range of variation in intensity appeared to make no difference to the readings, and the intensity was adjusted simply to give comfort in working.

IMMERSION REFRACTOMETRY.

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(Optics Department, National Physical Laboratory.)

MS. received, 26th May, 1920. Read and discussed, 10th June, 1920.

Introduction.

Two main methods of measuring to a high degree of accuracy the refractive properties of a specimen of glass are in general use. In the first method the specimen is cut in the form of a prism, usually of angle 60°. It is placed on the table of a spectrometer and the angle of minimum deviation for any given spectrum line is measured. The refractive index of the glass for light of the given wave-length is then calculated from the values of the angle of the prism and of the angle of minimum deviation. A modification of this method has been employed by Abbe and others*. The glass is cut in the form of a 30° prism and the incident beam of light, after refraction at the first face and normal reflection at the second face, is made to retrace its path, the observations being made by means of an auto-collimating arrangement. The second method depends on the determination of the path of a ray which enters a block of glass of known refractive index at grazing incidence at the surface of contact with the specimen, the block having a higher refractive index than the specimen to be measured. In this case the specimen of glass must be provided with one or two optically plane surfaces. The Abbe and Pulfrich refractometers are familiar examples of instruments which depend on this method of measurement. In addition to the methods mentioned there are of course others, such as those which depend on interference measurements; here again the provision of one or more optically plane surfaces is necessary.

None of these methods, however, can be applied directly when one wishes to determine the refractive properties of a specimen of glass which is unpolished, or is in the form of a lens, or is very small. It is frequently either inadvisable or impossible to grind and polish optically flat surfaces on a specimen on account of its being a component of some optical system which it is essential to keep intact, or because of its small size. In such cases the usual method adopted for measuring the refractive index for any given wave-length is to immerse the specimen in a transparent liquid having approximately the same index, and to vary the concentration of the liquid until the refractive indices of liquid and specimen for that wave-length are the same. The refractive index of the liquid is then measured by means of one of the ordinary methods. If the concentration of the liquid is so adjusted as to equalise the refractive indices for one wave-length, it will be found that the

* E. Abbe, *Neue Apparate zur Bestimmung des Brechungs- und Zerstreuungs-vermögens fester und flüssiger Körper*, Jena (1874); S. Czapski, *Z. f. Instrk.* 10 (1890), 361; C. Pulfrich, *Wied. Ann.* 45 (1892), 609.

indices differ for any other wave-length, on account of the different dispersions of the glass and liquid.

This dispersion difference and the difference in the temperature coefficients of refractive index in the case of solids and liquids give rise to very pretty colour effects when one warms a liquid in which a transparent powder having approximately the same index is mixed. The phenomena have been described by Christiansen*, Rayleigh†, and others.

Description of existing methods.

An immersion method of measuring the refractive index of glass in the form of a fine powder has been described by Christiansen‡. The powder is placed in a hollow glass prism of refracting angle 45° to 60°, which is filled with a mixture of carbon disulphide and benzene. When a homogeneous mixture has been obtained by means of stirring, the prism is placed on the table of a spectrometer, the collimator slit of which is illuminated by a sodium flame. The sodium line is seen sharply defined in the telescope. The powder gradually sinks to the bottom of the prismatic cell and the constitution of the mixture becomes different in different horizontal layers. As a result of this a band of light is seen in the field of view of the telescope. After some minutes, however, the powder collects at the bottom of the cell. One now sees two sharp lines, one due to refraction in the clear liquid and the other due to refraction in the saturated mixture. If v_1, v_2 and $v_1 + v_2$ are the volumes of the powder, the liquid, and the mixture respectively, and μ_1, μ_2, and μ are the corresponding refractive indices, then it follows from the equality of the optical paths that

$$(v_1 + v_2)\mu = v_1\mu_1 + v_2\mu_2.$$

If, now, the mixture is slightly altered by the addition of a small quantity of either benzene or carbon disulphide, and the refractive indices of the liquid and the mixture of powder and liquid are denoted by μ_2' and μ' respectively, then

$$(v_1 + v_2)\mu' = v_1\mu_1 + v_2\mu_2'$$

From these two equations it follows that

$$\mu_1 = \frac{\mu\mu_2' - \mu'\mu_2}{\mu - \mu' - \mu_2 + \mu_2'}.$$

Thus the refractive index of the powder may be determined from the results of measurements of the refractive indices of liquid and mixture for two slightly different concentrations of the liquid. Christiansen determined by this method the refractive indices of a specimen of crown glass for the lines *C*, *D*, and *F*. His mean results differed from those obtained by direct measurement by 1 to 5 units in the fourth decimal place.

The above method can only be applied to the case of very finely divided powders, for with coarser powders homogeneous mixtures cannot be satisfactorily obtained. Christiansen measured the refractive properties of coarser powders by a different

* C. Christiansen, *Wied. Ann.* 23 (1884), 298.

† Lord Rayleigh, *Phil. Mag.* 20 (1885), 358; *Scient. Papers*, vol. II, p. 433; *Nature*, 60 (1899), 64.

‡ C. Christiansen, *Wied. Ann.* 23 (1884), 298; 24 (1885), 439.

method, which is easier to apply, but does not yield quite so accurate results. The principle of the method is as follows. If one introduces glass powder into a suitable mixture of carbon disulphide and benzene, one can observe two colours, one of which consists of light which passes through without refraction, while the other is composed of all the other colours. The two colours are therefore complementary, the first being monochromatic and the second heterochromatic.

Christiansen in his experiments made use of two sensitive tints corresponding to monochromatic light of wave-lengths 530 and 460$\mu\mu$; the heterochromatic colours were reddish-violet and dirty yellow respectively. The concentration of the carbon disulphide and benzene mixture is gradually altered until the first sensitive tint is observed. The refractive indices of the liquid and the glass are then the same for the wave-length 530$\mu\mu$. The indices of the liquid for the lines C, D, and F are determined in the ordinary way and the index for the wave-length 530$\mu\mu$ is obtained by interpolation or with the aid of Cauchy's dispersion formula. The process is then repeated for the second sensitive tint. In this way the refractive indices of the glass for the wave-lengths 530 and 460$\mu\mu$ are determined. A specimen of flint glass was experimented with, and the values of the indices obtained by this method for the critical wave-lengths differed from those derived by direct measurement by 8 units and 1 unit in the fourth decimal place respectively.

The late S. D. Chalmers* employed a method of measuring the refractive indices of a lens, which consisted in immersing the lens in a liquid of approximately the same index and determining its focal length. If μ_1 is the index of the lens and μ_0 that of the liquid, the focal length F is given by

$$F = (\mu_1 - \mu_0)(R_1 - R_2),$$

where R_1, R_2 are the curvatures of the surfaces, considered positive when convex to the incident light, provided that the thickness of the lens is small in comparison with the radii of curvature. It is then only necessary to determine the values of F, R_1, and R_2 to the degree of accuracy that is required for the value of $\mu_1 - \mu_0$. A series of stable transparent liquids of suitable refractive index is used; for example cedar oil ($\mu_D = 1{\cdot}517$ approx.), and oil of cloves ($\mu_D = 1{\cdot}530$ approx.), are useful for the measurement of the indices of crown lenses. An average accuracy of about 5 units in the fourth decimal place was obtained with the method.

An immersion method of measuring the refractive constants of pieces of glass in the form of lenses and such like has been described by L. C. Martin†. The principle of this method is to immerse the specimen in a suitable liquid, such as carbon disulphide or mercury potassium iodide, contained in a prismatic cell resting on the prism table of a spectrometer, and to adjust the concentration of the liquid until the spectrum line, for which the refractive index of the specimen is to be measured, is seen in sharp focus. The angle of minimum deviation for the line in question is then determined and the refractive index calculated in the usual way. The diluent employed is alcohol, and the liquid in the cell is kept homogeneous by mechanical stirring. An average accuracy of about 1·4 in the fourth decimal place

* S. D. Chalmers, *Proc. Opt. Convention*, 1 (1905), 198.

† L. C. Martin, *Trans. Opt. Soc.* 17 (1916), 76.

was obtained by this method. Measurements were made on the *C*, *D*, and *F* lines of the spectrum, but the author states that when using mercury potassium iodide as the immersion liquid it was not easy to determine the values for the *F* line. The method possesses the distinct advantage of simplicity as regards apparatus. Any method, however, which depends on the determination of the sharpness of focus is liable to involve a greater degree of inaccuracy than one which depends, for example, on a coincidence setting.

An accurate method of immersion refractometry has been described by R. W. Cheshire*. It is based on the shadow method introduced by Foucault† for figuring optical surfaces. The image of a vertical straight edge, backed by a source of monochromatic light, is projected by a telescope objective into the plane of a second straight edge, which is so arranged as to cover about half the full aperture of the objective of an observing telescope. A rectangular cell containing the glass specimen immersed in a suitable liquid (mercury potassium iodide was employed) is placed near the first objective on the side away from the source of light. The strength of the liquid is varied until, on traversing the second straight edge across the objective of the observing telescope, which is focussed on the glass specimen, the whole field darkens simultaneously and uniformly. This only happens when the refractive indices of the glass and the liquid are the same. The index of the liquid is measured directly, the cell being cemented to the block of a Pulfrich refractometer. An average accuracy of 2 units in the fifth decimal place was obtained by this method in measurements on the sodium *D* line. It was found difficult to obtain accurate values of the indices for the hydrogen *C* and *F* lines owing to want of sufficient brightness.

In this connection it is of interest to note that the late Lord Rayleigh‡, in a paper dealing with the theory of Foucault's test, showed that the accuracy obtained with this method is of the order that might be expected. The method is essentially a very sensitive one and is, of course, much more accurate than any method which depends on the accuracy of focussing. It suffers, however, from the disadvantage of requiring a considerable amount of space, the distance between the cell and the observing telescope being about 5 metres. This involves the necessity of having two observers, one for determining the point when the refractive indices of the glass and the liquid are the same, and the other for measuring the index on the refractometer. This difficulty could possibly be minimised by the introduction of a mirror placed almost normal to the optical axis of the system, so as to bring the observing telescope into proximity to the refractometer telescope. Such an arrangement would, however, involve a certain amount of loss of light.

A method of determining the dispersion of a substance in the form of a powder is given by R. W. Wood§. The substance used for purposes of illustration is potas-

* R. W. Cheshire, *Phil. Mag.* 32 (1916), 409.

† L. Foucault, *Ann. de l'Observatoire de Paris*, 5 (1859), 197; *Recueil des travaux scientif.* Paris (1878), 232.

‡ Lord Rayleigh, *Phil. Mag.* 33 (1917), 161.

§ R. W. Wood, *Physical Optics*, (1914), 112.

sium fluosilicate formed in a solution of potassium chloride. The mixture is introduced into a hollow prism which is placed on the table of a spectrometer. When the precipitate has settled, the transmitted light, on being examined with a telescope, is seen to consist of a definite region of the spectrum, the mean wave-length of which can be determined. The clearness and narrowness of the band can be increased by screening off all light which passes through the clear liquid above the precipitate. The refractive index of the liquid for the particular wave-length in question can be measured in the usual way. The index of the mixture may be varied by adding potassium chloride or water, and thus the refractive indices of the precipitate for different wave-lengths can be measured. The transmitted light is not very pure, in the case of the fluosilicate, unless it has traversed a considerable thickness of the substance. More accurate results can therefore be obtained by putting the mixture into a long tube with a glass plate at each end, instead of into a hollow prism, and examining the transmitted light by means of a spectrometer. This method could be applied to the case of optical glass in the form of a fine powder, though the results obtained would probably not be of a high degree of accuracy, as the transmitted light would not be sufficiently monochromatic to allow of accurate determinations of its mean wave-length. The method would be applicable to the case of small fragments of glass which could be broken up, but not to the case of lenses which one did not wish to destroy. This criticism applies also to Christiansen's methods which have already been described.

One great disadvantage in most of the methods already described is the necessity for accurately adjusting the strength of the liquid so as to obtain equality of index for any given standard wave-length. Now anyone who has made such adjustments knows how laborious the process is. On approaching the balance point it is frequently found that, after adding a drop of the concentrated liquid or of the diluting medium in order, as the case may be, to increase or decrease the density of the mixture, the point of equality has been overreached. Even when one goes to the trouble of preparing for each wave-length two mixtures, one slightly denser and the other slightly less dense than the mixture of required refractive index, a considerable time is spent before the correct amounts of these mixtures has been added. Then again, even if the correct density of the immersion liquid has been attained, it may alter appreciably, due to evaporation at the surface, during the time occupied in making the necessary measurements of the refractive index.

Such difficulties are overcome by Ch. Fabry* in a method, a description of which has recently been published†. It depends on the measurement of small differences of refractive index by comparison with the index of a reference prism. The immersion liquid is contained in a cell with approximately parallel sides, and the reference

* Ch. Fabry, *J. de Phys.* 11 (1919), 11.

† My method also overcomes these difficulties, but in quite a different way. It was first used by me in the autumn of 1916 for the purpose of identifying the glasses of which a large number of lens components belonging to captured German optical instruments were made. Unfortunately, owing to pressure of other work, it has not been found possible to publish a description of the method till now. My attention was not drawn to Fabry's paper until the bulk of the present paper had been written.

prism and the specimen whose index is required are simultaneously placed in the liquid. A certain number of standard reference prisms is required, and in each case that one is chosen whose index is nearest to that of the specimen under examination. The index of the liquid is then determined by means of the deviation which the prism produces relatively to the ray which traverses the cell without passing through the prism. Fabry employs carbon disulphide as the immersion liquid. The cell is placed between a collimator and a telescope mounted on a divided circle, which reads to about one second. The faces of the cell are set normal to the collimator beam. The reference prism, whose refracting angle is 90°, is placed in the cell with its faces equally inclined to the incident beam. The observing telescope is provided with an objective of 40 cms. focal length and an adjustable eyepiece with millimetre graduations. The collimator slit is illuminated by means of a source of light giving a line spectrum. Now, if A be the angle and n the refractive index (corresponding to the wave-length of one of the lines) of the reference prism and N the index of the liquid, the angle of deviation D due to the prism is given by

$$D = 2(n - N)\tan A/2, \qquad \text{.....(1)}$$

if the difference between n and N is small.

Thus, if $A = 90°$,

$$N = n - D/2, \qquad \text{.....(2)}$$

an equation which gives N in terms of the known refractive index n and the deviation D, which is measured on the circle. If D is greater than 1°, it is advisable to use a more accurate formula, such as

$$N = n - \frac{1}{2}\sin D + \frac{1}{8n}\sin^2 D + \frac{1}{4n^2}\sin^3 D + \frac{15}{128n^3}\sin^4 D + \ldots, \qquad \text{.....(3)}$$

of which the third term is very nearly quite negligible. It gives correct values, where $A = 90°$, up to values of D of about 15°*. Fabry employs five reference prisms, cut by M. Jobin from glass made by Parra-Mantois. The constants of the glasses are as follows:

	C	D	F
Light boro-silicate crown ...	1·49464	1·49693	1·50230
Ordinary crown	1·51744	1·52008	1·52643
Light barium crown ...	1·57155	1·57450	1·58155
Dense flint	1·59998	1·60450	1·61575
Extra dense flint	1·64045	1·64585	1·65945

With these prisms one can measure all indices between 1·47 and 1·67 with the difference $N - n$ not exceeding ± ·03 and in consequence the deviation D not exceeding ± 3°·5.

In order to obtain the value of N accurate to one in the fifth decimal place, the following errors of measurement, etc. must not be exceeded:

Measurement of D: ± 4 seconds.

Error of angle A: 1 minute.

Equality of inclination of prism faces to incident beam: 30 minutes.

Angle between walls of cell: 1 degree.

Angle between incident beam and normal to cell walls: 1 degree.

* H. S. Uhler (*Amer. Journ. Sci.* 49 (1920), 143) discusses the general case where the angle A may have any value, and gives alternative rapidly convergent series.

The method of obtaining the refractive indices of a specimen in the form of a lens is as follows. The strength of the liquid is roughly adjusted so that its index is very nearly equal to that of the lens for a given radiation. The position of the telescope draw-tube, for which the line in question is sharply focussed, is determined when the reference prism alone is immersed in the liquid. The lens is then immersed in the path of the beam of light, and the position of the draw-tube is altered by an amount x so as to bring the line into sharp focus again. At the same time the deviation D is measured. The composition of the liquid is then slightly altered and new values of x and D are measured. Having obtained two or three pairs of values, including positive and negative values of x, a curve is drawn with x and D as coordinates. From this curve, which is practically a straight line, the value of D which corresponds to $x = 0$, that is, to equality of index of lens and liquid, can be deduced. The index of the lens can then be obtained from the formula (3). Fabry finds that the error of focussing the telescope is of the order of $\pm$ 1 mm. and theoretically deduces the fact that, in order to obtain for refractive indices an accuracy of one in the fifth decimal place, the difference in thickness of the centre and of the edge of the effective portion of the lens must at least reach the value 6000λ. Thus a difference in thickness of 4 mm. is sufficient to ensure this accuracy throughout the spectrum.

The method as applied to a specimen in the form of a prism is somewhat similar. The prism is immersed in such a way that its edge is perpendicular to the edge of the reference prism. A wire is stretched across the collimator slit, its image appearing as a black point. The vertical deviation d of this image is measured by means of a micrometer eyepiece, the motion of which is parallel to the length of the slit, and at the same time the horizontal deviation D caused by the reference prism is determined. The simultaneous measurements of D and d are made for two or three different strengths of the immersion liquid, and a curve is drawn through the points corresponding to the values obtained. From this curve, which is practically a straight line, the value of D which corresponds to $d = 0$ may be obtained, and then the index of the specimen can be deduced, as in the former case, from formula (3). Fabry found that the method gave results correct to 1 or 2 units in the fifth decimal place in the case of a specimen of quartz, the angle between adjacent faces of which was 120°. He does not indicate for what wave-length the measurement was made.

Fabry's method can also be applied to the cases of irregular fragments with curved faces (such as rods of glass or beads of borax fused to platinum wire) and transparent substances in the form of powders with coarse grains.

The method is in many respects a great improvement on those that have already been described, but it is doubtful whether the degree of accuracy claimed, namely one unit in the fifth decimal place of refractive index, will be attained in general practice. Although it might appear from the list of allowable errors of setting, etc. given on p. 200, that this degree of accuracy could be obtained with quite an approximate fulfilment of the ideal geometrical conditions, it must be remembered that it is the cumulative effect of all these errors that is to be taken into account. This

really involves much narrower limits than those given by Fabry. Further, it should be noted that there should be no lateral play in the telescope draw-tube. Thus it appears that, in order to obtain an accuracy of one unit in the fifth decimal place, somewhat rigid conditions must be fulfilled in practice. It would be interesting to know what accuracy can be attained with ordinary specimens of glass. The only result given is for a piece of quartz, with which an accuracy of one or two units in the fifth decimal place was found. It is not fair, however, to draw conclusions from this one result as the angle utilised for the deviation was very large, namely 120° (cf. footnote on p. 215).

It is interesting to note that the principle of immersion refractometry is used in mineralogy for determining the refractive indices of mineral fragments*. If a small specimen be immersed in a liquid and a low power microscope be focussed on an edge (preferably a sharp one) of the specimen, the edge is barely visible, provided that the refractive indices of the specimen and liquid are equal. If the indices are different, it is found that, on racking the microscope away from the specimen, a bright line appears near the edge inside the medium of higher refractive index; on racking the microscope in the other direction, the bright line appears just inside the medium of lower index. This bright line is usually referred to as the "Becke line"; by making use of it one can determine whether the immersion liquid has a higher or lower refractive index than the specimen. The strength of the liquid can then be varied until the line no longer makes its appearance, and the refractive index can be determined by one of the usual methods. The results obtained are not very accurate, but are usually sufficiently correct for the purposes of identification. The Becke line is also made use of for determining whether one constituent in a rock section has a higher or lower index than an adjoining constituent.

A more accurate method of measuring the refractive indices of mineralogical specimens has been described by F. E. Wright†. It is really a combination of Foucault's shadow method with microscopic measurements. Two knife-edges (safety razor blades are used) are mounted horizontally, being fixed to a vertical bar which is attached to the side of the stage support. The upper blade is between the microscope objective and the slide, being in the image plane of the lower blade. When the upper blade is moved into the position where its edge just touches the edge of the image of the lower blade, a mineral grain in the slide appears bright on a dark field, if its index is different from that of the liquid in which it is immersed. The edges of the grain which appear bright become dark when the upper blade is withdrawn. This reversal adds to the sensitiveness of the method. Monochromatic light is used, and one determines between which two of the spectrum lines of wave-length 546, 560, 578, and 588$\mu\mu$ the refractive indices of the mineral and the liquid coincide, the strength of the liquid being previously adjusted to come within this range. One can then calculate approximately (to within about $\pm$ ·001) what the refractive index of the mineral grain for the sodium *D* line should be

* Henry A. Miers, *Mineralogy*, (1902), 259, 260.

† F. E. Wright, *Journ. Washington Acad. Sci.* 4 (1914), 389.

This can be checked by immersing the grain in a specially prepared liquid having exactly this index. The author states that a monochromatic illuminator has sometimes been found useful for ascertaining approximately the wave-length for which the index of the grain coincides with that of the enveloping liquid.

Description of present method.

The main principle of the present method of immersion refractometry is as follows. For the sake of simplicity it may be assumed that it is required to find the refractive index of a lens for the wave-length corresponding to the *C* line in the hydrogen spectrum. A monochromatic illuminator, which has been carefully calibrated, is used as the source of light. The strength of the immersion liquid is first of all roughly adjusted so that a balance in refractive index is obtained for a wave-length in the neighbourhood of the *C* line. In order to keep the density of the liquid uniform it is stirred continuously by means of a mechanical stirrer. The wave-length of the light from the monochromatic illuminator is varied until the particular wave-length is determined, for which the refractive indices of lens and liquid are equal. The index of the liquid *for the C line*, or the corresponding angle reading on the refractometer or goniometer, is then quickly determined. The density of the liquid is next slightly altered and, when a steady state has again been reached, the new wave-length for which the indices are equal is found, the corresponding index of the liquid for the *C* line being again measured. The process is repeated until a number of such points in the neighbourhood of *C* (preferably on both sides of *C*) are found, and the refractive indices, or angle measurements, are plotted against the corresponding wave-lengths for which the balance points were obtained. On drawing a curve through the points thus plotted one can immediately determine what the refractometer angle reading, and therefore the refractive index, should be for exact equality of index for the *C* line. The process may then be repeated in the neighbourhood of any other wave-length for which the index is required. No mention has so far been made of the way in which the balance points are determined, but a number of alternative methods will be described later.

The chief point of the process consists in the substitution of a variation of wave-length for a variation of the strength of the immersion liquid. The method has the obvious advantage over any of the existing methods of immersion refractometry that it is much quicker and therefore less liable to errors due to variations in the density of the immersion liquid during observations. The wave-length of the light can be altered very much more rapidly than the strength of the liquid. A number of balance points can be determined very quickly, and if the curve through these points should not prove to be satisfactory, a number of extra points can be found with little extra trouble.

Fabry's method also obviates the necessity of fine adjustments of the strength of the immersion liquid. As compared with the present method, however, it has the disadvantage of requiring the use of a set of reference prisms, the indices of which must be known very accurately. This of course may in some cases not

be a very serious objection, but it will probably be found that in general the present method is a more convenient one, especially in cases where it is necessary to make a large number of determinations of refractive index over a large range of glasses.

It may be pointed out that great accuracy of calibration of wave-length in the monochromatic illuminator is requisite, since a small error in wave-length determination causes an appreciable error in the value found for the refractive index.

Apparatus.

The apparatus required in connection with the method consists of a glass cell, a monochromatic illuminator, a refractometer or goniometer for measuring the refractive index of the immersion liquid, and suitable observing apparatus for determining the balance points.

Cell.

The cell employed for holding the liquid in which the specimen of glass is immersed is the one used by R. W. Cheshire in his experiments*. It is rectangular in shape, the internal dimensions being $5 \times 2 \times 1{\cdot}5$ cms. The base plate is made of extra dense flint glass, $\mu_D = 1{\cdot}74$, its thickness being 2 mm. When used with a Pulfrich refractometer the cell can be employed for measurements of refractive index of liquids up to $\mu_D = 1{\cdot}66$. The parallelism of the upper and lower faces of the base plate is correct to within 5 seconds of angle. The upper edge of the base plate which is nearer to the source of light used with the refractometer is chamfered. This prevents any deposition of dust, etc. at the bottom of the cell from interfering with the path of light and thus causing the edge of the spectrum line, as seen in the refractometer telescope, to be ill-defined. In such experiments as the ones to be described it would be advantageous to use a wider cell than the one which was employed, as the full aperture of the refractometer block could then be utilised, and further, the greater volume of liquid which could be used with it would tend to reduce the rate of variation of refractive index of the liquid due to evaporation at the free surface.

It may be mentioned here that some difficulty was experienced at first with the cell, as the ordinary cements employed for building it up were soon disintegrated by the mercury potassium iodide, which was the liquid used. Satisfactory results, however, were obtained by cementing the cell with Canada balsam. The cell remained intact over long periods, care being always taken to wash it thoroughly after each occasion when it was used. For future experiments it might be advisable to make use of cells built up by means of the method of heat treatment described by Parker and Dalladay†.

In order to keep the immersion liquid as homogeneous as possible during experiments, it is stirred by means of a rotary glass stirrer actuated by a small motor. When possible, the top of the cell is covered by a glass plate so as to reduce evapora-

* R. W. Cheshire, *Phil. Mag.* 32 (1916), 409.

† R. J. Parker and A. J. Dalladay, *Trans. Faraday Soc.* 12 (1917), 305; *Phil. Mag.* 33 (1917), 276.

tion at the free surface of the liquid, the plate having a hole in it for the stem of the stirrer to pass through. In the case of small lenses or small fragments of glass a good plan is to cement the glass to the end of the stirrer, and thus to use the specimen itself as the stirrer. A somewhat similar method may be applied in the case of larger pieces of glass, the specimen being caused to move fairly rapidly to and fro across the path of light. Such devices, however, can only be employed in the case of some of the methods used for detecting the balance points. The chief difficulty in keeping the liquid homogeneous occurs in the case of large specimens which project from the liquid. The evaporation at the surface of contact causes the formation of "drops" of liquid of greater density, and these fall to the bottom of the cell. This difficulty is not, however, a very serious one, if the liquid is kept well stirred.

Monochromatic illuminator.

A Hilger constant deviation spectrometer was used as a monochromatic illuminator. The eyepiece was removed and a vertical slit placed in the focal plane of the eyepiece. In some of the methods employed for determining the balance point of refractive index this slit was replaced by a fine vertical spider line traversed at the centre by two horizontal parallel lines, the purpose of which is merely to fix the position of the centre of the vertical line. The advantage of using a fine line instead of a slit in certain spectroscopic measurements has been dealt with by J. Guild*.

No advantage would be obtained by using Hilger's specially designed monochromatic illuminator, for the increased illumination due to the short focus lenses would be counterbalanced by the decreased length of the spectrum.

Instead of the wave-length scale supplied with the Hilger constant deviation spectrometer a uniform millimetre scale was used, and the instrument was carefully calibrated in terms of this scale. It is only necessary to determine the scale readings corresponding to the spectrum lines for which the refractive indices of the specimen are required. The advantages of using a uniformly divided scale are two-fold. In the first place it is much easier to determine fractions of a scale unit, and in the second place, if the scale readings for the balance points are plotted against the angular readings on the refractometer, the curves obtained are straight lines; this would not be the case if the ordinary wave-length scale were employed.

As source of light for the monochromatic illuminator a "Pointolite" tungsten arc was employed. This gives a very bright spectrum and enables measurements to be made with ease on the portion of the spectrum in the neighbourhood of the hydrogen *F* line.

Refractometer.

All the measurements of refractive index of the immersion liquid were made with a Pulfrich refractometer, the cell containing the liquid being placed in position on the block of the refractometer and kept there during the experiments. Optical

* J. Guild, *Proc. Phys. Soc.* 29 (1917), 328.

contact between the base of the cell and the refractometer block was obtained in the usual way by the use of a drop of α-monobromonaphthalene. The present method of immersion refractometry could, of course, be equally well applied to the case in which the refractive index of the liquid is determined by means of an ordinary spectrometer, a prismatic cell being used instead of a rectangular one.

There is one point to be noted in connection with the measurement of refractive indices for the sodium D lines. In making direct determinations with a glass specimen on the block of a Pulfrich refractometer it is the edge of the D_1 line (that is, the line of greater wave-length) on which one makes a setting. In the case of the liquid employed in the experiments, however, the spectrum is reversed on account of the relative dispersions of the liquid and the glass of the refractometer block. Thus in measuring the refractive index of the liquid one sets on the edge of the other sodium line, namely D_2. In order, therefore, to co-ordinate the results obtained by the immersion method with those derived from direct readings on the specimens under examination, small corrections have to be made to the former so as to give the values of the indices for the D_1 line. The manner in which these corrections are applied will be described later.

As source of light for the refractometer a hydrogen tube of the H form was used in the "end-on" position, the tube being renewed with fresh hydrogen and then exhausted before each measurement. The auxiliary apparatus employed was shown at the Optical Society's Exhibition on the 11th of January, 1917, and is described in the Society's *Transactions**. It consists essentially of a home-made electrolytic generating hydrogen apparatus which works automatically. In order to obtain the sodium lines from the hydrogen tube, the tube is only exhausted to a pressure of about 3 or 4 cms. of mercury. When a fairly heavy discharge is passed for a short time the capillary of the tube becomes heated and, being made of soda glass, it emits bright lines, the secondary hydrogen spectrum forming a good background for illuminating the cross-lines in the eyepiece of the refractometer telescope. The use of this method obviates the necessity of having an additional source of light for the sodium lines.

Measurements of the refractive indices of specimens were made for three lines, namely, the C and F hydrogen lines and the sodium D_1. No determinations of refractive index for the hydrogen G' were made on account of the large percentage absorption of violet light by the mercury potassium iodide. In order to identify a specimen, however, it is not essential to measure its index for G'; the data for three lines, such as C, D_1, and F, are quite sufficient.

Auxiliary Apparatus.

The additional apparatus required for the experiments will be described in detail in the case of each method employed. In building up the optical systems required for detecting small differences in refractive index special attention has been paid to the question of compactness. In some cases a certain loss in sensitive-

* *Trans. Opt. Soc.* 18 (1917), 21 S.

ness has been involved, but this has been more or less counterbalanced by the convenience of having the apparatus taking up as small a space as possible. The positions from which the balance point and the refractive index of the liquid are observed have been brought into close proximity, wherever possible, so that one person can make the two sets of observations with ease and with a minimum loss of time.

Method (1). The first method that was employed for determining the wavelength for which the refractive indices of liquid and specimen are equal is illustrated in plan in Fig. 1. The light from a "Pointolite" tungsten arc P is focussed on the first slit S_1 of the monochromatic illuminator I by means of a lens L_1. An image of the vertical spider line stretched across the second slit S_2 of the illuminator is formed at S_3 by the lens L_2, a reflecting prism R_1 being used to deflect the beam of light

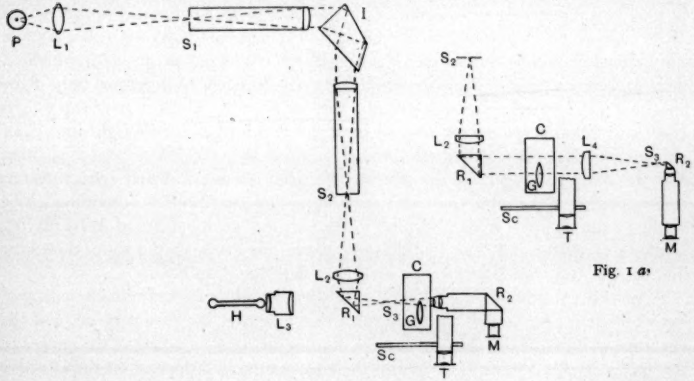


Fig. 1 *a*.

Fig. 1.

through a right angle. C represents the cell containing the liquid in which the specimen of glass G is immersed. The mechanical stirrer used for mixing up the liquid in the cell is not shown. Optical contact is made between the base plate of the cell and the block of the Pulfrich refractometer by means of a drop of α-monobromonaphthalene, or, in the case of extra dense flints, mercury barium iodide. T represents the observing telescope, Sc the vertical scale, and L_3 the condensing lens of the refractometer. H is the hydrogen vacuum tube which is used as the source of light for the measurement of the refractive index of the liquid. The telescope T during the experiment is below the horizontal plane containing the axis of the optical system for measuring the balance point of refractive index. A low-power microscope M is focussed on S_3 through the liquid. A 3-inch objective is used in the microscope in order to get sufficient working distance between S_3 and the microscope for the introduction of the cell. For convenience of observation

a reflecting prism R_2 is employed to deflect the beam of light in the microscope through a right angle. In this way the eyepieces of the refractometer telescope and the microscope are brought close together. The prism R_1 is mounted on a small table attached to the refractometer. It is rotated out of the position shown in the diagram when the wave-length has been adjusted for equality of index of liquid and glass, in order that the refractometer measurement may be made.

The method of measuring the refractive index of the glass specimen G for, say, the hydrogen C line is as follows. The concentration of the liquid is first of all roughly adjusted until its refractive index for C is nearly equal to that of the specimen. This can be rapidly done, before the cell is placed in position on the refractometer block, by altering the strength of the liquid until the image of an object (preferably illuminated by red light) seen through the specimen and the liquid very nearly coincides with the image seen through the liquid alone. When the cell is in position, the specimen is mounted in such a way that it intercepts part of the beam of light which passes through the liquid. In general, therefore, two images of the spider line will be seen in the microscope; the one which is formed by the light which traverses the specimen and the liquid will be fairly well defined only if the specimen is regular in shape (such as a lens or prism). The drum of the monochromatic illuminator is now rotated until the two images are brought into coincidence and the scale reading s is noted. The prism R_1 is then swung out of position and the scale reading r on the micrometer drum of the Pulfrich refractometer, corresponding to the refractive index of the liquid for the C line, is determined. The reading s can be made from a position in the neighbourhood of T and M by placing a reflecting prism and lens combination above the drum of the monochromatic illuminator, the scale being illuminated by a flash-lamp bulb.

The concentration of the liquid is next altered slightly by introducing a drop of liquid of somewhat higher or lower refractive index, as the case may be, and the observer waits until the mixture is made homogeneous by stirring. Two images of the spider line will again be seen and their separation will depend on the effective angle of the edge of the specimen used and on the difference of index of the liquid caused by the addition of the drop. In order to get as great a separation as possible it is therefore advisable to adopt such an orientation of the specimen as to present a maximum effective angle in the path of the beam. The wave-length of the light is now altered until the scale readings s and r are determined, the latter corresponding to the refractive index of the new mixture *for the C line*. The process is repeated for two or three different concentrations of the liquid, the refractive index being varied by small steps.

The pairs of values of s and r thus obtained are now plotted and a curve is drawn through the points. From this curve the value r which corresponds to the scale reading s for the wave-length of the hydrogen C line is determined; this value of r then gives the angular reading on the refractometer which would have been obtained if a mixture with the same refractive index as the specimen for the C line had been made up. The reason for plotting values of s and r, and not the values of the wave-lengths and the refractive indices, is, as has already been pointed out,

that the curve thus obtained *is a straight line*, provided that the scale of the monochromatic illuminator is an evenly divided one. On this account only a few sets of readings are necessary for each determination of refractive index.

A similar process to the above is applied in the case of the other wave-lengths employed, namely those corresponding to the sodium D_1 and hydrogen F lines. In the case of the former the refractometer settings are made for the sodium D_2 line, as has already been mentioned. The simplest method of making the necessary corrections may be described as follows. The refractive indices of mercury potassium iodide solutions of strengths corresponding to the range of glasses to be tested are measured for the lines C, D_2, and F. Let r_C, r_{D_2}, and r_F be the emergent angles, as measured on the refractometer, corresponding to these indices. The values of r_{D_2} and $r_C - r_F$ for each refractometer block used are plotted and from the curves thus obtained the value of $r_C - r_F$ for any given value of r_{D_2} can be determined. In order to find the corresponding refractometer reading for D_1, that is r_{D_1}, use may be made of any of the well-known dispersion formulae which involve two constants, such, for example, as the simplified form of Cauchy's formula; this may be written

$$r = r_0 + A/\lambda^2,$$

where r_0 and A are constants. From this formula we obtain

$$r_C - r_F = A\,(\nu_C{}^2 - \nu_F{}^2) \text{ and } r_{D_1} - r_{D_2} = A\,(\nu_{D_1}{}^2 - \nu_{D_2}{}^2),$$

where $$\nu = 1/\lambda.$$

Thus $$r_{D_1} = r_{D_2} + \frac{\nu_{D_1}{}^2 - \nu_{D_2}{}^2}{\nu_C{}^2 - \nu_F{}^2} \cdot (r_C - r_F).$$

If the numerical values of ν for the lines C, D_1, D_2, and F are introduced, we obtain

$$r_{D_1} = r_{D_2} + \cdot 003\,(r_C - r_F)$$

approximately.

From this formula, therefore, we can deduce what the refractometer reading should be for the sodium D_1 line, if the reading for the D_2 line and the value of $r_C - r_F$ for the corresponding strength of immersion liquid are known. The value of the correction that is thus made may amount to nearly one minute of angle, which corresponds to a difference in refractive index of about one in the fourth decimal place.

If, instead of using a dispersion formula, the slope of the dispersion curve at the wave-lengths of the sodium lines be taken as equal to the slope of the straight line joining the points corresponding to the wave-lengths of the C and F lines, the factor ·0035 is obtained for the second term of the above equation. For most purposes this factor is sufficiently accurate. If, however, a more accurate correction is required, one may employ the slope of the line joining the points corresponding to the wave-lengths of the sodium D_2 and the helium yellow lines. This would involve the determination of a number of readings for the helium line; the difference between the correction thus obtained and the one which the dispersion formula gives would at most be equivalent to about one unit in the fifth decimal place. It is not likely that such a degree of refinement will be necessary in connection with

methods of immersion refractometry, but the point is of importance in the measurement of the refractive indices of highly dispersive liquids by means of the Pulfrich refractometer.

Method (2). The second method that was adopted for determining the balance points is similar to the one just described, but is somewhat more sensitive. The optical system employed is shown in Fig. 1 *a*. The image S_3 of the vertical spider line stretched across the second slit S_2 of the monochromatic illuminator is formed by two lenses L_2 and L_4, one on either side of the cell C, the beam of light which passes through the liquid being rendered parallel by the lens L_2. One advantage of using this system rather than the one shown in Fig. 1 is that a higher power microscope may be employed, since its working distance is not limited by the interposition of the cell between S_3 and the microscope objective. A small right-angle prism R_2 is attached to the objective of the microscope M so as to bring the eyepiece of the latter into close proximity to that of the refractometer telescope T.

Both of these methods (but especially the latter) are very convenient for observing the balance point, particularly when the specimen under examination is in the form of a lens or prism. Attention may again be directed to the fact that,

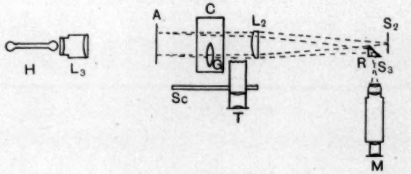


Fig. 2.

in order to obtain the greatest accuracy with any given specimen, the greatest effective angle of the specimen should be utilised, so as to give a maximum deflection of the image formed by the light which passes through the glass and the liquid. Thus, for example, in the case of a flat lens the specimen should be mounted edgewise in the beam of light, for with such an orientation it is practically equivalent to a thin prism of large angle.

There are one or two ways in which method (2) may be modified so as to give a somewhat higher degree of accuracy. The use of a much higher power microscope in combination with a finer spider line is not of any great advantage, since the intensity of illumination might be cut down too much, and this would be a drawback, especially in the determinations of indices for the F line. For the lens L_4 (Fig. 1 *a*), however, a longer focal length objective could be used, and in order to keep the system compact a pentagonal prism could be substituted for the right-angle prism R_2; if the pentagonal prism were fairly large, a considerable portion of the optical path between the lens and its focus would be in the prism. Another modification, whereby the beam of light is made to pass twice through the specimen, and thus give double the deviation, is shown in Fig. 2. The second slit S_2 of the mono-

chromatic illuminator is placed at the focus of the lens L_2, so that a parallel beam of light is caused to pass through the cell C. This beam is reflected back by a plane mirror A, slightly inclined to the beam, and after again traversing the cell is brought to a focus at S_3, a small right-angle prism R being inserted so as to deflect the light through 90°. The spider line image formed at S_3 is then observed by means of a microscope M. The mirror A may be mounted on a table which can be swung out of position when the refractometer measurements are being made, or it may be fixed in such a way that the beam of light from the hydrogen tube H passes underneath it. By placing the mirror at a small angle to the beam, the reflected light is brought to a focus at the side of the slit S_2, thus involving less loss of light than if a half-silvered mirror were used instead of the prism R, and, further, the images formed by light reflected from the walls of the cell do not disturb the main image which is to be observed.

Method (3). Another method, which is convenient to employ in the case of lenses and prisms, the edges of which form fairly large effective angles, is illustrated in Fig. 3.

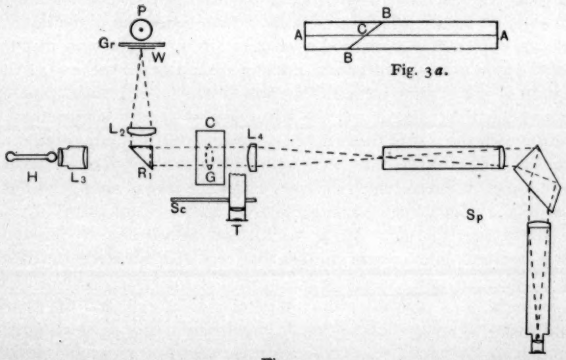


Fig. 3.

Fig. 3*a*.

A piece of ground glass *Gr* is illuminated by a "Pointolite" tungsten arc *P* and a very fine wire or spider line *W* is mounted horizontally on the other side of the ground glass. An image of this wire is formed across the slit of a spectrometer *Sp* by means of two lenses L_2 and L_4, the wire being at the focus of the former. The cell C is mounted in the path of the parallel beam of light between L_2 and L_4, the general arrangement of apparatus being much the same as in the previous method. The specimen is mounted in such a way that its edge intercepts the upper half of the beam of light which passes through the liquid in the cell. Before the specimen is immersed in the liquid one sees in the spectrometer a spectrum traversed horizontally by a fine dark line AA (Fig. 3 *a*), corresponding to the point on the slit where the image of the wire *W* falls. If now, the specimen is placed in position, another dark

line *BB* will be observed. This line, which is straight if the specimen is a prism and approximately straight if it is a lens, makes an angle with the line *AA* depending on the effective angle of the specimen. The point *C* where these two lines intersect corresponds then to the wave-length for which the refractive indices of the liquid and the specimen are equal. This method is a very pleasant one to work with, but it does not give such accurate results as the previous ones, unless the angle of the specimen is fairly large. With specimens of small angle it is sometimes rather difficult to determine the point *C* with accuracy as the region round *C* is filled with systems of fringes.

Method (4). Another method of observing the balance point is similar to the one employed by R. W. Cheshire and is specially convenient in the case where the specimen to be examined is in the form of an irregular fragment. The general arrangement of apparatus is the same as that shown in Fig. 1 *a*, except that, instead of using a fine vertical spider line across the second slit of the monochromatic illuminator, the slit is narrowed and a fine wire or spider line is made to coincide at S_3 with the image of the slit formed by the light which traverses the immersion liquid alone. No observing microscope is required; on placing one's eye behind the wire at S_3 no light reaches it unless the refractive indices of the liquid and the specimen are different. The method of observation is, therefore, to alter the wave-length of the light until the specimen suddenly disappears in the field of view. This modification of the shadow method does not, of course, give such accurate results as Cheshire's method, but it has the advantage of greater compactness and, in combination with the rest of the process, of greater available intensity.

Method (5). One of the most accurate methods of observing the balance point is to make use of an interferometer system*. For the sake of simplicity this may be built up on the plan of the Rayleigh interference refractometer†. Experiments carried out by my colleague, Mr J. Guild, and myself some years ago with a Michelson interferometer system showed that very accurate determinations of the equality of refractive index could be obtained. The method, however, necessitates the employment of an optically accurate, parallel-sided cell, and the system must be mounted in such a way that it is free from vibration. For most purposes, therefore, the increased sensitivity does not compensate for the more rigorous conditions required. The method would, however, be extremely useful for detecting and measuring small variations of refractive index in specimens of irregular shape.

Results.

In order to test the accuracy that can be obtained with the present method of immersion refractometry, three small prisms were experimented with, the glasses being a boro-silicate crown, a baryta light flint, and a dense flint. For purposes of comparison the refractive indices of these specimens were determined by direct measurement on the Pulfrich refractometer. For observing the balance points

* The problem has recently been dealt with by C. Barus, "The Interferometry of Reversed and Non-reversed spectra," Part II, *Carnegie Institution of Washington Publication*, 249 (1917), 95.

† Lord Rayleigh, *Phil. Mag.* 33 (1917), 161.

method (2) was employed. The following measurements of the scale readings on the monochromatic illuminator (s) and the Pulfrich refractometer (r), which were obtained in the case of the B.S.C. specimen for the C line of the hydrogen spectrum, will serve to illustrate the linear relation between s and r.

s	r
738·5	35° 43′·6
736·9	35° 45′·0
728·8	35° 52′·2
725·4	35° 55′·1
717·4	36° 2′·3

If these values be plotted, it will be found that the points lie very nearly on a straight line, from which the value of r corresponding to the scale reading 724·8 for the C line is obtained, namely, 35° 55′·9. The reading obtained by direct measurement was 35° 55′·1.

The following table gives the results of the refractive index measurements for the three specimens for the lines C, D_1, and F. The last column represents the errors as compared with the direct measurements.

Specimen	Line	μ (immersion method)	μ (direct method)	$\delta\mu$
B.S.C.	C	1·50576	1·50584	− ·00008
	D_1	1·50821	1·50822	− ·00001
	F	1·51371	1·51377	− ·00006
B.L.F.	C	1·54751	1·54756	− ·00005
	D_1	1·55062	1·55068	− ·00006
	F	1·55825	1·55828	− ·00003
H.F.	C	1·61747	1·61750	− ·00003
	D_1	1·62234	1·62240	− ·00006
	F	1·63470	1·63478	− ·00008

The average error, therefore, is about 5 units in the fifth decimal place. A still higher degree of accuracy might be obtained by slight modifications of the method, such as have been suggested on p. 210.

Theoretical discussion of accuracy obtainable.

We may now discuss theoretically the accuracy to be expected from this method of determining the balance point. Consider the case (Fig. 4) of a prism P

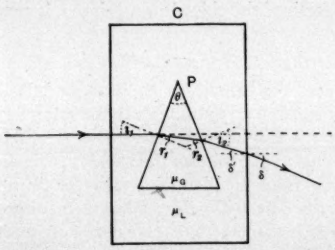


Fig. 4.

immersed in a parallel-sided cell C filled with liquid, the base of the prism being parallel to the incident ray, which is normal to the sides of the cell. Let the refractive indices of the prism and the liquid for a given wave-length be μ_G, μ_L respectively, where $\mu_G > \mu_L$ the difference $\mu_G - \mu_L$ being very small. Further let θ be the angle of the prism, i_1, i_2 the angles of incidence and emergence at the prism faces, r_1, r_2 the angles of refraction in the prism, and δ', δ the deviations of the ray after emergence from the prism and from the cell, respectively. Then, since the angles of deviation are very small,

$$\delta = \mu_L \delta' = \mu_L \{(i_1 + i_2) - (r_1 + r_2)\}.$$

Also $\mu_L \sin i_1 = \mu_G \sin r_1$ and $\mu_L \sin i_2 = \mu_G \sin r_2$.

Therefore
$$\begin{aligned}(\mu_G - \mu_L) \sin r_1 &= \mu_L (\sin i_1 - \sin r_1) \\ &= 2\mu_L \sin \frac{i_1 - r_1}{2} \cos \frac{i_1 + r_1}{2} \\ &= \mu_L (i_1 - r_1) \cos \theta/2,\end{aligned}$$

since the difference $i_1 - r_1$ is very small and $\sin \frac{i_1 - r_1}{2}$ may be replaced by $\frac{i_1 - r_1}{2}$, and $\frac{i_1 + r_1}{2}$ by i_1 or $\theta/2$.

Similarly $(\mu_G - \mu_L) \sin r_2 = \mu_L (i_2 - r_2) \cos \theta/2$.

Therefore

$$\delta = \frac{\mu_G - \mu_L}{\cos \theta/2} (\sin r_1 + \sin r_2) = \frac{2(\mu_G - \mu_L)}{\cos \theta/2} \sin \frac{r_1 + r_2}{2} \cos \frac{r_1 - r_2}{2}$$

$$= 2(\mu_G - \mu_L) \tan \theta/2, \qquad \text{.....(1)}$$

since r_1, r_2 are each very nearly equal to i, and $\cos \frac{r_1 - r_2}{2}$ is approximately unity.

If now a parallel beam of rays falling normally on the first side of the cell is brought to a focus by an objective after traversing the prism and cell, the displacement d of the image in the focal plane will be given by

$$d = f \tan \delta,$$

where f is the focal length of the objective, or, since δ is very small, $d = f\delta$. Thus from equation (1)

$$d = 2f(\mu_G - \mu_L) \tan \theta/2. \qquad \text{.....(2)}$$

Now, in the experiment the breadth of the image of the line as formed at S_3 (Fig. 1 *a*) was approximately ·0075 mm. It may be assumed that with the magnification of the microscope employed, namely about 30, a relative displacement of the two line images equal to one third of the breadth of the line could be detected. Thus in order to obtain from equation (2) a value for the minimum difference in refractive index that can be detected by this method we may insert the value ·0025 mm. for d. The prisms experimented with had angles of approximately 12°·5 and the objective employed had a focal length of about 20 cms. Putting these values into

equation (2) we obtain as the minimum difference in refractive index the value ·000057*.

Another source of possible error is in connection with the calibration of the monochromatic illuminator. Now the accuracy with which settings can be made on the monochromatic illuminator is about ·1 of a small division on the uniform scale. Further, from the slopes of the straight lines, from which the values of the refractive indices are interpolated, it follows that ·1 of a small scale division is equivalent to an angle on the refractometer scale of from ·09 to ·14 of a minute depending on the type of glass. This corresponds roughly to an accuracy of one unit in the fifth decimal place of refractive index, provided the Pulfrich refractometer is accurate to that degree. It follows, therefore, that the mean accuracy obtained in the experiments is of the order that one would expect from theoretical considerations.

SUMMARY

The paper contains brief descriptions of the chief methods employed for measuring the refractive indices of specimens which cannot be measured directly.

A new method of immersion refractometry is described. This depends essentially on the principle of substituting for the usual method of varying the strength of the immersion liquid a process which involves a few determinations of the wave-lengths for which the refractive indices of liquid and specimen are equal, corresponding to different strengths of the liquid.

A number of alternative methods of observing the balance point of refractive index are given. Results obtained by one of these methods were found to give for the C, D_1, and F lines a mean error of about 5 units in the fifth decimal place, though this error may be somewhat reduced by modifications of the optical systems employed. The method was employed with great success during the war in determining the types of glass used for the lens components of a number of captured German optical instruments.

In conclusion, the author desires to express his thanks to Mr J. Guild, A.R.C.Sc. for many valuable suggestions, and to Mr R. J. Trump, B.A., B.Sc., for his kind assistance in the earlier part of the work.

DISCUSSION

Mr T. Smith said that the author was to be congratulated for developing a method, available for a number of wave-lengths, which was very accurate and quick, and involved the use of inexpensive apparatus. It was advantageous to have a method, such as had been described, which could be carried out by a single observer. The process had been found extremely useful during the war. He congratulated the author on the inclusion in the paper of a very complete bibliography of the subject.

* For the case of the quartz specimen, for which Fabry gives results (p. 202), the detectable difference in refractive index would be about 2 units in the sixth decimal place, as the prism angle was 120°.

Mr L. C. Martin communicated the following remarks.

We are indebted to Dr Anderson for the description of his very beautiful experimental method. To complete the bibliography of the subject the papers by Chalmers (*Opt. Conv.*, 1905), and Carl Barus (*Pub. Carnegie Inst.*, 1916 and 1917, "The Interferometry of Reversed Spectra") might be mentioned. Chalmers seems to have anticipated certain points in Fabry's method, while Barus has used the interferometer. It occurs to me to suggest that an observation of the displacement of interference fringes in a type of optical system similar to the Rayleigh gas refractometer, where the immersed specimen is introduced into one limb, would give a sensitive indication, although probably not more so than the "knife edge" method. Where a monochromatic illuminator and refractometer are not available I still consider my own method capable of very good results. At the time of publication I was working with an iron spectrum. It was easy to tell the region of equal refractivé index from inspection of the spectrum and the refractive index of the liquid could be roughly checked in this way while altering the mixture, and afterwards delicately changed by evaporation. Dr Anderson's ingenious solution of the interpotation problem saves a certain amount of experimental work, but by working as suggested above there is no need to anticipate any difficulty or waste of time in the spectrometer method.

The author communicated the following reply.

I have now inserted references to the methods used by Chalmers and Barus, and would like to thank Mr Martin for drawing my attention to them. The application of a system similar to the Rayleigh gas refractometer has already been referred to in the text.

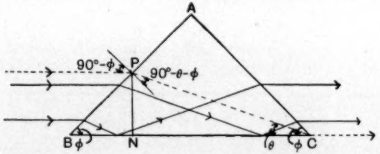
THE INVERTING PRISM

By Instructor-Commander T. Y. Baker, R.N., B.A.

MS. received, 4th June, 1920. Read, 10th June, 1920.

In many instances a prism is used in an instrument to invert an image by a single reflection without alteration of the direction of the light. A very common form of this is the ordinary prism, with angles 45°, 45°, and 90° which is used so that light enters parallel to the hypotenuse face and after refraction at one of the short sides is reflected in the hypotenuse and subsequently emerges parallel to its original direction. It is quite clear that exactly the same effect could have been obtained with any prism in which the sides AB and AC are equal and that there is no necessity to limit the shape to one in which the angles at B and C are 45°.

The extreme ray which can get through the prism in this way is shown in pecked lines and what we may call the numerical aperture of the prism (i.e. the



ratio $BC : PN$) varies with the base angles and the refractive index of the glass. As in certain circumstances as large an aperture as is possible may be required, the following note discusses the relationship of base angle with refractive index in order to give maximum aperture. If the base angles B and C are each ϕ and the angle PCB is θ the refraction equation becomes

$$\cos\phi = n\cos(\phi + \theta).$$

We also obtain $BC/PN = \cot\phi + \cot\theta$.

Hence, to fulfil the required conditions $\cot\phi + \cot\theta$ must be a minimum with the condition

$$\cot\phi \operatorname{cosec}\theta = n(\cot\phi\cot\theta - 1).$$

Writing $\cos\theta = x$, we have

$$\cot\phi = n\sqrt{1-x^2}/(nx-1),$$

and

$$\frac{n\sqrt{1-x^2}}{nx-1} + \frac{x}{\sqrt{1-x^2}}$$

is then to be a minimum.

This leads to the following equation for the determination of x

$$x^3 - 2nx^2 + x + n - 1/n = 0.$$

This is a cubic equation which gives as solutions

(i) a value between $-1/n$ and $-1/2n$,

(ii) „ „ $1/n$ and 1,

(iii) „ „ n and $2n$.

It is clear that the only possible practical value is the second root for which $x = (7n + 6)/(10n + 3)$ is closely approximate.

The values of θ and ϕ for the values of $n = 1{\cdot}5,\ 1{\cdot}55,\ 1{\cdot}60,\ 1{\cdot}65,\ 1{\cdot}70$ are tabulated below:

n	θ	ϕ	Aperture ratio base/height	Aperture ratio with 45° prism	Aperture ratio with 30° prism
1·50	23° 36′	31° 57′	3·89	4·29	3·90
1·55	24 25	32 43	3·76	4·11	3·78
1·60	25 9½	33 23	3·65	3·94	3·68
1·65	25 51	33 59	3·55	3·81	3·59
1·70	26 29	34 32	3·46	3·69	3·51

These tabulated values show that no very large improvement is possible but that a prism of angles 30°, 30°, 120° will, in all cases, be better than one of the ordinary 45° pattern.

EXHIBITS

FLUORESCENT PHENOMENA

20th May, 1920

Mr W. Shackleton exhibited certain fluorescent effects that can be obtained with the ultra-violet radiations transmitted by Messrs Chance's ultra-violet glass, and demonstrated how different specimens of Crookes' glass absorb those radiations. The source of light employed was a carbon arc behind a quartz condensing lens.

Mr F. Harrison Glew showed that fluorescent effects could be obtained without the use of a quartz condensing lens. The source of light which he used was a 200 candle-power filament lamp placed in an ordinary dark-room lantern with a window of Chance's special glass. He showed that a candle fluoresces and suggested that lubricants used in optical instruments might contain some volatile constituent similar to the fluorescing constituent in candles.

DISCUSSION

Mr F. E. Lamplough said that much better results could be obtained by using a powerful mercury vapour lamp as a source of light, as this was very rich in ultra-violet radiations. He stated that the ultra-violet glass was being employed as a new instrument of medical research, particularly by Prof. Russ. The fluorescent effect of ultra-violet light on the skin had been photographed and may prove of considerable value in diagnosis. When the eye was turned in the direction of intense rays of ultra-violet transmitted by the dark glass the fluorescence of the lens of the eye produced the effect of a blue haze surrounding everything and most people experienced fatigue and headache. The interposition of a Crookes' glass between the source of light and the eye immediately cut off the blue haze and gave a sense of relief.

In response to large numbers of enquiries for samples of ultra-violet, uranium, and Crookes' glasses for demonstration purposes, Messrs Chance Brothers are getting ready a small set of glasses illustrating some of the phenomena of ultra-violet glass and fluorescence, and these sets will be available shortly.

PROCESS OF MOULDING SMALL LENSES

10th June, 1920

Mr A. Wood gave a demonstration of a quick method of moulding small lenses. The two portions of the mould are held at the ends of the prongs of an ordinary pair of pliers. A strip of glass is heated in a Bunsen flame and pressed between the two halves of the mould when at the correct temperature. The projecting portions of the lens thus formed are knocked off and the lens is placed on a hot plate for

the purpose of annealing. The only process that requires to be applied subsequently is that of edging. Mr Wood stated that the method occurred to him in the early days of the war as a solution of the very difficult problem of producing small lenses of short focal length, by means of unskilled labour, in sufficient quantities to meet the very large demand. It was found that, after a few days' practice, one girl was able to produce by the moulding process more lenses in a given time than six girls had hitherto been able to do by the usual method of grinding and polishing. A high degree of accuracy is not claimed for these lenses, but it is thought that the process may be of use in enabling us to cope with the foreign competition with which we are likely to be faced in the near future, especially in the cheaper class of optical instruments.

A telescope ($\times$ 30), fitted with a moulded eye lens and giving good definition, was exhibited.

DISCUSSION

Mr F. F. S. Bryson asked if the surface of the moulding tool required to have a protecting layer, or was reliance placed on the formation of soot from the flame.

Mr P. F. Everitt said he understood the tools were made of steel, and enquired what grade of finish was required and what the life of a tool was.

Instructor-Commander T. Y. Baker asked if the curves of the mould were exactly those of the finished lens or whether any appreciable alteration in curvature took place during cooling.

Mr Wood, in reply, stated that no protecting layer was required on the surface of the tool, and that no soot was formed, as the glass was heated in a Bunsen flame. The mould was ground and required to be re-surfaced on a glass-working tool with fine emery (wet) after about 2 or 3 hours. The moulding process gave the necessary curves.

SAMPLES OF OPTICAL GLASS

10th June, 1920

Mr H. C. Rands exhibited a number of samples from special glass meltings produced by the Derby Crown Glass Co., Ltd. He said that the firm had now made on a commercial scale fluor crown glass having the values $\mu_D = 1{\cdot}4947$ and $\nu = 70{\cdot}0$, and that it was now in a position to produce phosphate crown glass in large quantities, if there were sufficient demand for it from opticians.

DISCUSSION

Mr F. F. S. Bryson said he thought that the fluor crown glass which was exhibited would prove to be of considerable use to optical manufacturers.

REVIEW

Optical Projection. Part I. The Projection of Lantern Slides. By LEWIS WRIGHT, 5th edition, re-written and brought up-to-date by RUSSELL S. WRIGHT, M.I.E.E. Pp. vi + 87; 50 figures. (London: Longmans, Green & Co., 1920.) Price 4*s.* 6*d.* net.

This small hand-book deals only with the projection of lantern slides; in the second part, which will be published later, the author will deal with the demonstration of opaque and microscopic objects, scientific phenomena, and accessory apparatus, including cinematograph projection. The present volume is intended to assist the ordinary users of optical lantern apparatus. It contains descriptions of practically every kind of light source employed for projection purposes and numerous useful hints with regard to their manipulation. It would, however, have been advisable for the author to have devoted rather less space to the description of the simpler and more obsolete forms of lamp and to have given somewhat fuller accounts of the more modern and more generally used types. For example, the Ediswan "Pointolite" lamp, which forms an extremely compact, steady, and reliable source of light and is specially suited for small lecture theatres and halls where an electric supply is available, receives but scanty notice. The optical system of a lantern is very clearly explained with the aid of simple diagrams, and various forms of lantern bodies and slides are described. Two chapters are devoted to a brief account of a number of accessories, such as lantern boxes, stands, reading lamps, screws and screen stands, and the book concludes with various hints regarding the general manipulation of a lantern. The text is well written and the illustrations are numerous and of excellent quality. It is to be regretted that no index of contents is provided; it may, however, be the intention of the author to include an index, dealing with both parts, in the second volume. The book can be recommended as a useful compendium of information which can easily be understood by lantern manipulators who do not possess any great technical knowledge.

J. S. A.

ABSTRACTS

"On tracing Rays through an Optical System." By T. SMITH, B.A. *Proc. Phys. Soc.*, 32 (1920), 252.

An exact method of tracing rays is developed which may be considered a continuation of that suggested in "Some Generalised forms of an Optical Equation" [*Trans. Opt. Soc.*, 20 (1918), 23]. The formulae take the shape

$$\text{change on refraction} = \text{paraxial effect} + \frac{\text{first order aberration}}{D},$$

where D is a denominator less than 1, which is expressed in a number of forms. It can be calculated with a sufficient degree of accuracy by using a short table of cosines in terms of sines, thus dispensing with the usual trigonometrical tables. When a calculating machine is used a saving of time is effected by the adoption of the new equation.

T. S.

"A Graphical Method of finding Axial Conjugate Points in an Optical System." By Dr A. KÜHL, *Central-Zeitung f. Opt. u. Mech.*, 41 (1920), 199.

If x and x' are the distances of conjugate points in an optical system, measured from the foci F and F' in the object and image spaces respectively, and f and f' are the corresponding focal lengths, the relation

$$xx' = ff'$$

holds. The author gives a graphical method of determining the positions of conjugate points, if f, f', and the positions F, F' are known. A circle is drawn with FF' as diameter and two lines XX, $X'X'$ are drawn parallel to the axis at distances f, f' respectively from it. The distance x is laid off on XX from the foot of the normal from F and the point thus obtained is joined to F, the line being produced to cut the circle in O. The point O is then joined to F' and the line produced to cut $X'X'$. The distance of the point of intersection from the foot of the normal from F' is then the distance x' of the conjugate point.

J. S. A.

"The Hensoldt-Hildebrand Reading Microscope." By K. LÜDEMANN, *Z. f. Vermessungswesen*, 49 (1920), 314.

The paper describes a new reading microscope for use with the verniers on divided circles. Between the objective and the eyepiece of the microscope is a Hensoldt roof prism which deflects the light through 70°. The objective end can rotate in the ring which holds the microscope in position, so that two verniers situated at opposite ends of a diameter of the divided circle can be read from approximately the same position. Thus an observer does not require to move round the circle in order to read both verniers. The microscope has a magnification of 14 diameters as against about 9 diameters in the case of the ordinary types in use. The author gives the results of a number of observations on the relative degree of accuracy obtainable with the new and old types, and shows that the new microscope is more accurate under different conditions of illumination of the verniers and scale.

J. S. A.

"A Variable Power Telescopic Rifle Sight." By Dr R. RICHTER, *Central-Zeitung f. Opt. u. Mech.*, 41 (1920), 200.

The instrument described is called the "Panskopar" and is manufactured by Voigtländer of Brunswick. The magnification can be varied continuously from $4\frac{1}{2}$ to $7\frac{1}{2}$ diameters. The erecting system consists of two components, one of which remains fixed in position, while the other is moved along the axis in order to vary the power. The total range of motion is obtained by a quarter rotation of an adjustment ring at the centre of the telescope. The maximum variation in focus is equivalent to about three-quarters of a diopter, so that it lies within the range of accommodation of the eye. The eye-point is about 8 cms. behind the eyepiece, the aperture of which has to be made relatively large, namely about 35 mm., in order to get a sufficiently large field of view.

J. S. A.

"The Melting of Glass." By F. WEIDERT and G. BERNDT, *Z. f. techn. Phys.*, No. 6 (1920), 121.

Glass belongs to the group of substances which have no definite melting points. The change from the solid to the fluid state extends over a considerable range of temperatures, in many cases some hundreds of degrees; the magnitude of the range depends on the definition of the two limiting temperatures. The authors suggest the terms "deformation temperature" and "flow temperature" for the two limits. The former refers to the point at which the glass begins to soften and the latter the point at which the glass is in a fluid state. These temperatures depend on the duration of the heating process and on the size of the specimen. A heating period of half-an-hour and a cube, the length of whose side is 25 mm., are taken as the standard conditions. A number of experiments were carried out with specimens of optical glass. Each cubical specimen was half embedded in slag, one of the diagonals of the cube being vertical, and was heated fairly rapidly in an electrical furnace. The temperature was measured by means of a thermo-couple placed near the projecting apex of the specimen. The flow temperature could be determined to within $\pm$ 10°. The results of experiments are given in the following table, measurements

Type of glass	Analogous Schott type	Flow temperature in degrees C. for ½ hour	2 hours	6 hours
Kron 510/634	O. 144	850	815	775
Kron 516/640	O. 3832	810	795	780
Flint 549/461	O. 378	740	725	685
Barion 573/575	O. 211	910	885	860
Barint 580/538	O. 722	845	805	785
Barion 590/612	O. 2122	845	830	795
Barion 609/589	O. 2071	870	835	820
Flint 613/369	O. 118	730	695	680
Barion 614/564	O. 2994	840	815	800
Barint 626/393	O. 748	780	730	685
Flint 649/338	O. 102	660	645	630

being made for heating periods of ½, 2, and 6 hours. The first column gives the designation of the glass as used by the Sendling Optical Works at Zehlendorf, near Berlin. The number 510/634 is intended to show that the glass has the values $\mu_D = 1{\cdot}510$ and $\nu = 63{\cdot}4$. The second column gives the analogous type of glass in the catalogue of Schott of Jena.

J. S. A.

NOTICES

The Secretary of the Department of Scientific and Industrial Research has sent the two following notices:

Professor Sir John Cadman, K.C.M.G., D.Sc., University of Birmingham, Mr W. B. Hardy, Sec. R.S., Gonville and Caius College, Cambridge, and Professor Sydney Young, D.Sc., F.R.S., Trinity College, Dublin, have been appointed by an Order of Council dated 24th June, 1920, to be members of the Advisory Council to the Committee of the Privy Council for Scientific and Industrial Research.

Professor John Bretland Farmer, D.Sc., M.A., F.R.S., Imperial College of Science and Technology, has been appointed by an Order of Council dated the 28th day of August, 1920, to be a member of the Advisory Council to the Committee of the Privy Council for Scientific and Industrial Research.

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